Estimating rigidity index (I_R) based on CPT measurements

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ABSTRACT: The rigidity index (I_R) is defined as the ratio of the shear modulus (G) to the undrained soil strength (s_u) and is a critical parameter for estimating the consolidation coefficient (c_h) using cone data. I_R has also been shown to influence the stresses and pore pressures that develop around an advancing cone. The most accurate method to determine I_R is through the use of advanced laboratory tests on high quality samples, though simplified methods do exist (such as chart solutions based on stress history and plasticity). Since cone tests and in-situ correlations are commonly used, a new method is proposed that only requires use of cone data; no complimentary laboratory tests or field sampling is required. A database of laboratory and cone tests from several clay sites was used to evaluate existing methods and to develop a new method for predicting I_R using only cone test data.

1 INTRODUCTION

The routine use of cone penetration tests (CPT) as an in-situ site investigation tool has increased due to the ability to obtain a variety of measurements (q_c, f_s, u_2, and V_s) (Lunne et al. 1997). Significant research has been devoted to understanding the mechanism of cone penetration and developing theoretical and analytical methods to properly model soil displacement around the advancing cone (e.g. cavity expansion, strain path, bearing capacity theories, finite element/difference models) (Yu et al. 2000, Lu et al. 2004). These methods can be used to provide fairly accurate predictions of expected plastic flow mechanisms during cone penetration. Many researchers have shown that the plastic zone formation during cone penetration has a primary dependence on the soil's rigidity index (I_R) (Vesic 1972, Teh 1987, Teh & Houlsby 1991, Schnaid et al. 1997, Yu et al. 1998), with I_R originally defined as the ratio of the soil's shear modulus (G) to the undrained shear strength (s_u). The plastic failure zone that develops during cone penetration was shown to increase as I_R increases (Teh 1987), and consequently influences the generation of excess pore pressure (Δu) and the coefficient of consolidation (c_h).

A simplified and more reliable method for estimating I_R is desirable for geotechnical engineering applications. The use of laboratory data to determine I_R is complex and expensive while existing empirical and analytical methods are based on small, specific data sets (not originally intended for CPT interpretation). Chart solutions are available using easily identifiable parameters, overconsolidation ratio (OCR)

and plasticity index (PI) (Keaveny 1985; updated by Mayne 2007), but even these require acquisition of soil samples. Since the CPT is commonly used, a method that can estimate I_R based solely on CPT measurements and correlations from a high quality database is desirable and can be used to supplement laboratory test data. A large database of clay sites (including both advanced laboratory tests and CPT data) was used to evaluate empirical trends to develop a new method to estimate I_R.

2 RIGIDITY DEPENDENCE

The dependency of I_R on G and s_u requires careful evaluation of both parameters. The value selected for s_u should represent the average strength mobilized around an advancing cone. The most appropriate test method is the anisotropic (K_0) consolidated undrained triaxial compression test ($CAUC$) (Keaveny 1985, Schnaid et al. 1997, Yu et al. 2000). The selection of an appropriate shear modulus is a primary challenge since G is a function of strain level, aging effects, and various other factors (Wroth et al. 1979, Schnaid et al. 1997). The initial shear modulus, G_{max}, typically represents the tangent modulus at low strains (< 0.01%), while a secant modulus is used for larger strain levels and G decreases with increasing strain level (Houlsby & Wroth 1991, Mayne 2007).

The shear modulus at 50% mobilized strength (G_{50}) is often selected for use in determining I_R. Researchers suggest that use of G_{50} represents the average response of the engaged soil volume (Konrad & Law 1987, Schnaid et al. 1997). While there is some evidence that G_{50} works better for overconsolidated clays than for normally consolidated clays (Schnaid et al. 1997, Jamiolkowski 2003), the use of G_{50} remains the best method to provide reasonable estimates of I_R for clays.

The coefficient of consolidation (c_h) can be estimated through interpretation of CPT pore pressure dissipation tests (Teh & Houlsby 1991, DeJong & Randolph 2012, Krage et al. 2014). Teh and Houlsby's (1991) analytical solution shows that c_h is proportional to $I_R^{0.5}$ and Robertson et al. (1992) also showed that t_{50} increases with increasing I_R. The possible range of I_R (50-500 for clays) creates significant uncertainty in estimating c_h.

3 ASSEMBLED DATABASE FOR EVALUATION OF RIGIDITY INDEX

A database of laboratory tests and CPT soundings for a range of clays was assembled in order to evaluate different methods for estimation of I_R. $CAUC$ tests from high quality block (Laval and Sherbrooke samplers) or fixed piston (Osterberg) samples obtained at various sites were used in order to determine I_R. All samples are from marine and estuarine deposits, with PI ranging from 20 to 80 and OCR up to 4. A summary of the test sites and samples is presented in Table 1.

All CPT data used in the database was obtained using a 10 cm^2 cone pushed at the standard rate of 2 cm/s. All G_{max} values reported by the original studies were either seismic CPT V_s measurements or determined using laboratory bender element measurements. OCR values are based on in-situ estimates, except for cases where the OCR of the lab specimen was reported and was similar to in-situ estimates.

The compiled database was evaluated to identify any apparent trends in G and s_u. Investigated trends included OCR, PI, peak undrained strength, ratio of G_{50} to G_{max}, and laboratory measured I_R. When appropriate residuals were quantified using:

$$Residual = \ln\left(\frac{x_{predicted}}{x_{actual}}\right) \quad (1)$$

where $x_{predicted}$ is the predicted value and x_{actual} is the reference/measured value. Strength is normalized using the SHANSEP framework where S [$=(s_u/\sigma'_{vo})_{NC}$] can be approximated by 0.32 +/- 0.03 for CAUC tests and m can range from about 0.8 for low to medium sensitive clays and between 0.9 and 1.0 for structured clays (Ladd 1986, Kulhawy & Mayne 1990, El Hakim 2005). As evident in Figure 1, the best fit for the current database is *S* and *m* values of 0.33 and 0.75 respectively. The range of *PI* used in this study is consistent with previous work by Ladd (1986) who suggested that *S* is independent of *PI*. Data also indicated that the shear strain required to reach the peak undrained strength increases with OCR (Broussard 2012). This observation is consistent with Ladd (1986), who also documented that for mechanically OC clays the shear strain to failure increases with OCR for triaxial compression tests.

Table 1. Summary of Clay Database

Clay Site	Location	Deposit	OCR	PI	LL	Sensitivity	Reference
Boston Blue Clay	Newbury, Mass (US)	Glacial Marine	2.5-4.1	19-21	45-49	Low (2-4) and Extra Quick (16-32)	Landon (2007)
Onsoy	Fredrikstand, Norway	Uniform Marine	1.2-1.5	22-38	55-71	Low to Medium (4-9)	Landon (2007)
Burswood	Perth, Australia	Estuarine	2.8-2.9	32-54	71-99	Low to High (3-14)	Landon (2007)
Leda	Gloucester, Canada	Marine	1.2 -1.9	27-38	53-62	Highly Sensitive to Quick (18 and 15)	Landon (2007)
Ariake	Hizen-Kashima, Saga Prefecture, Japan	Marine	1.2-1.6	65-70	110-120	High (20-40)	Shibuya et al. (2000) & Tanaka (2000)
Bangkok (NNH)	Non Ngu Hao Site, Thailand	Marine	1-1.4	46-80	73-120	Low to Medium (3-8)	Shibuya and Tamrakar (2003) & Shibuya et al. (2000)
Bangkok (AIT)	Asian Institute of Technology Test Site, Bangkok, Thailand	Marine	1.1	24-40	91-117	Low to Medium (up to 8)	Shibuya and Tamrakar (2003) & Shibuya et al. (2000)
Bothkennar	Science and Engineering Research Council Site, UK	Estuarine (Post Glacial)	1.6	30-40	56-76	N/A	Nash et al. (1992) & Tanaka (2000)

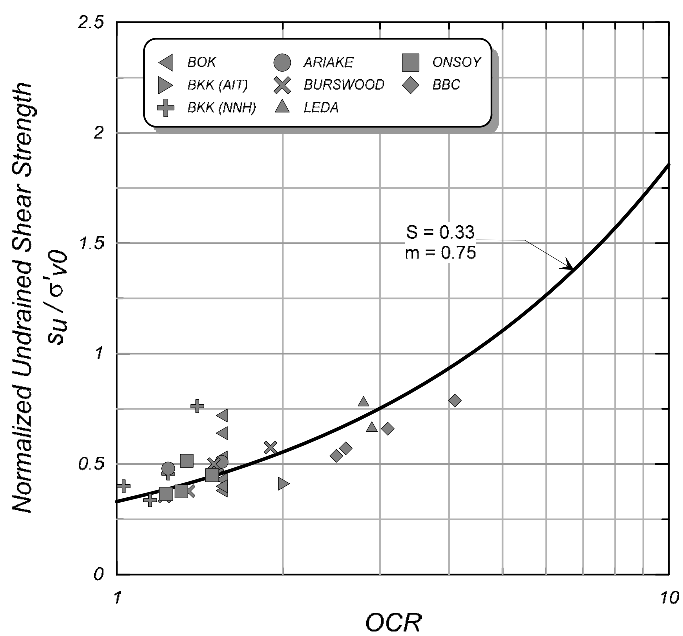

Figure 1. Normalized undrained shear strength obtained from lab tests

The ratio of G_{50}/G_{max} was computed by obtaining G_{50} from the stress-strain behavior of *CAUC* triaxial test data and using seismic CPT measurements or laboratory bender elements for G_{max}. Figure 2 suggests that G_{50}/G_{max} is independent of *OCR* for monotonic tests, resulting in a G_{50}/G_{max} ratio of 0.26 for this database.

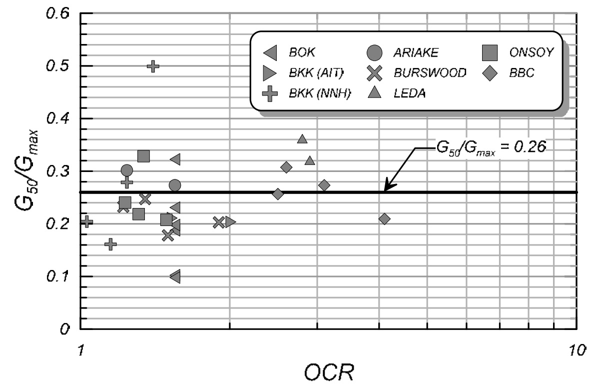

Figure 2. Relationship of G_{50}/G_{max} vs. *OCR* for the database soils. Reference line of G_{50}/G_{max} = 0.26 is selected for use in empirical relationships.

I_R tends to decrease with increasing *OCR* and *PI* as shown in Figure 3. This is consistent with observations by previous researchers (Keaveny 1985, Wroth et al. 1979). G_{50} is dependent on total vertical stress and G_{50}/G_{max} is approximately constant for *OCR*s 1-5. These empirical observations are useful to evaluate existing methods and develop new methods for determining I_R. More observations of database trends are available in Broussard (2012).

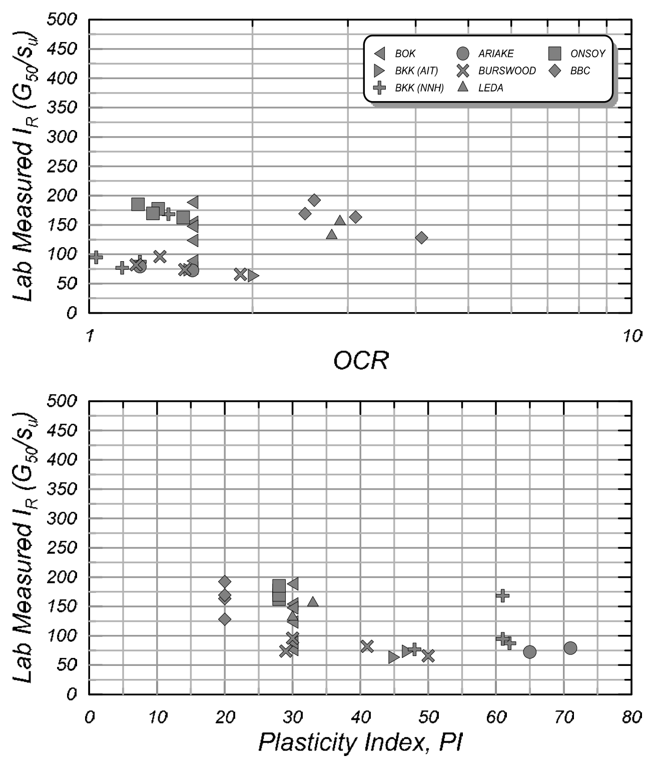

Figure 3. Relationship between laboratory CAUC triaxial test based rigidity index (I_R)

4 EMPIRICAL METHODS FOR DETERMINING RIGIDITY INDEX

The primary method used to estimate I_R currently is based on the work of Keaveny and Mitchell (1986). In their work analyzing the performance of shallow foundations they suggested that I_R decreases with an increase in both *OCR* and *PI*. In order to best fit *CAUC* data from five test sites, Keaveny (1985) suggested the following relationship between *K* and I_R.

$$I_R = \frac{\frac{K}{2}}{2*(1+\nu)} = \frac{K}{6} = \frac{E_u}{6*S_u} \tag{2}$$

Equation 2 assumes the G_{50}/G_{max} ratio to be 0.5, since G_{max} equals $E_u/3$. Test results from this database suggest that G_{50}/G_{max} is closer to 0.3. The original data from Keaveny (1985) presented in Figure 4 shows a lack of high *OCR* data and significant scatter between the data points, which suggests that I_R can range from 75 to 200 for an NC soil with a *PI* of 30. Mayne (2007) provided an approximated the Keaveny (1985) chart solution with the following relationship:

$$I_R \approx \frac{\exp(\frac{137-PI}{23})}{[1+\ln\left(1+\frac{(OCR-1)^{3.2}}{26}\right)]^{0.8}} \tag{3}$$

Various analytical methods also exist for estimating I_R. Methods like those developed by Wroth and Houlsby (1991), Kulhawy and Mayne (1990), and Mayne (2007) incorporate *OCR* in their development, but the relationships are omitted from this study due the inability to evaluate these methods with information contained in the assembled database.

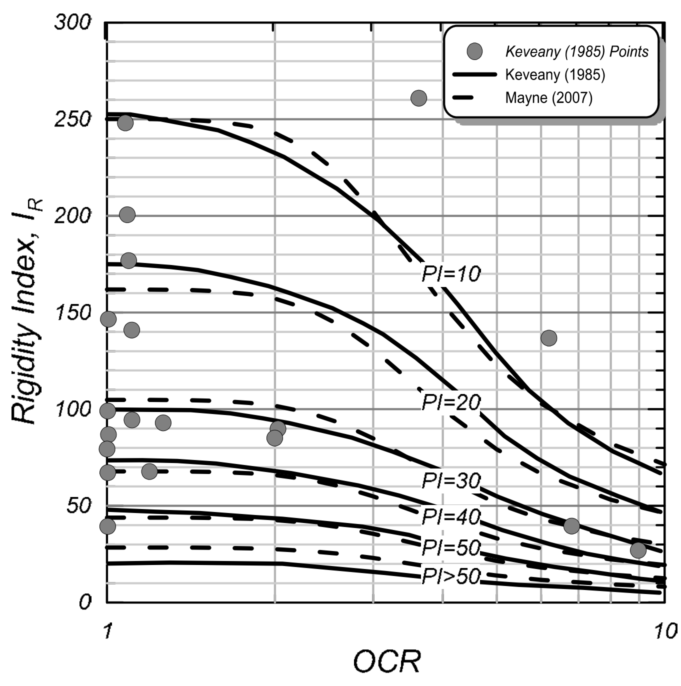

Figure 4. Keaveny (1985) chart solution along with original test results and Mayne (2007) approximation of the chart.

5 DEVELOPMENT OF NEW METHODS TO PREDICT RIGIDITY INDEX

Due to the difficulty in obtaining high quality samples, cost of performing lab tests, and natural site variability, it is desirable to predict I_R using in-situ methods like the CPT. The undrained strength required for estimating I_R is considered using two different approaches. The first approach (Method A) utilizes the actual laboratory measured values in the database to estimate I_R. This method does not circumvent the use of laboratory data, but is used to provide a direct comparison with empirical methods. The development of new simplified methods for estimating I_R is enabled through use of several database trends and is proposed as follows:

$$I_{R\_Method\ A} = \frac{G}{s_u} = \left(\frac{G_{50}}{G_{max}}\right) * \left(\frac{G_{max}}{\sigma'_{vo}}\right) * \left(\frac{\sigma'_{vo}}{s_u}\right) \quad (4)$$

where Method A assumes G_{50}/G_{max} is 0.26 (empirical estimate from Figure 2), G_{max} is obtained from seismic CPT tests, and s_u is obtained from laboratory tests.

The second approach (Method B) uses only seismic CPT data. A normalized strength relationship is used (assuming values for S and m) and the preconsolidation stress is estimated using a factor of 0.33 (Mayne 2007). Method B enables estimation of I_R solely from CPT data using the following symbolic and functional equations:

$$I_R = \left(\frac{G_{50}}{G_{max}}\right) * \left(\frac{G_{max}}{\sigma'_{vo}}\right) * \left(\frac{1}{\left[\frac{s_u}{\sigma'_{vo}}\right]_{NC} * OCR^m}\right) = \left(\frac{G_{50}}{G_{max}}\right) * \left(\frac{G_{max}}{\sigma'_{vo}}\right) * \left(\frac{1}{\left[\frac{s_u}{\sigma'_{vo}}\right]_{NC} * \left[\frac{\sigma'_p}{\sigma'_{vo}}\right]^m}\right) \quad (5)$$

$$I_{R\_Method\ B} = 0.26 * \left(\frac{G_{max}}{\sigma'_{vo}}\right) * \left(\frac{1}{0.33 * \left[\frac{0.33*(q_t - \sigma_{vo})}{\sigma'_{vo}}\right]^{0.75}}\right) \quad (6)$$

If V_s measurements are unavailable for a given site, correlations can be used to estimate V_s from CPT measurements for use in determining G_{max}. Wair et al. (2012) lists several methods to estimate V_s from CPT measurements. Note that G_{max} would then be estimated from V_s and density correlations, increasing the scatter of estimating G_{max}.

6 EVALUATION AND VALIDATION OF METHODS

The Keaveny (1985) chart solution, Method A, and Method B were evaluated against the measured I_R values in the experimental database. Figure 5 presents a comparison between measured and predicted I_R for each method. These results show that Keaveny (1985) consistently under predicts I_R while Method A (which uses the laboratory based s_u and empirical G_{50}/G_{max} relationship) and Method B (which uses empirical correlations and CPT data) perform well. Residual values (Fig. 6) are not biased toward *OCR* or *PI* and are mostly between +/-0.5.

It is important to note that Methods A and B are both based on fewer data points than the Keaveny (1985) method. There seems to be less scatter between data points when using Keaveny (1985) than when using Methods A or B. Keaveny's solution, however, tends to under predict I_R, resulting in larger residual values compared to Methods A or B. Overall Method B performs comparably, if not better, than Method A and the Keaveny chart solution and only requires the use of CPT data and correlations.

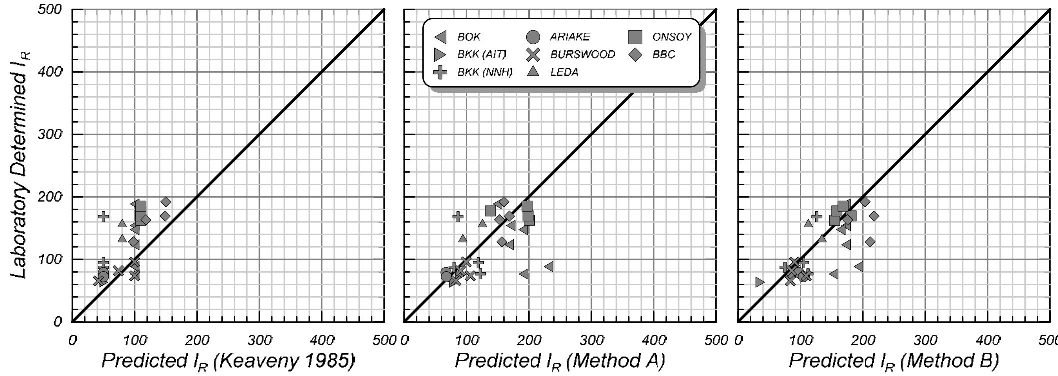

Figure 5. Comparison of predicted to measured rigidity index values using (left to right): Keaveny (1985), Empirical G_{50}/G_{max} based Method A, and Empirical G_{50} / G_{max} based Method B.

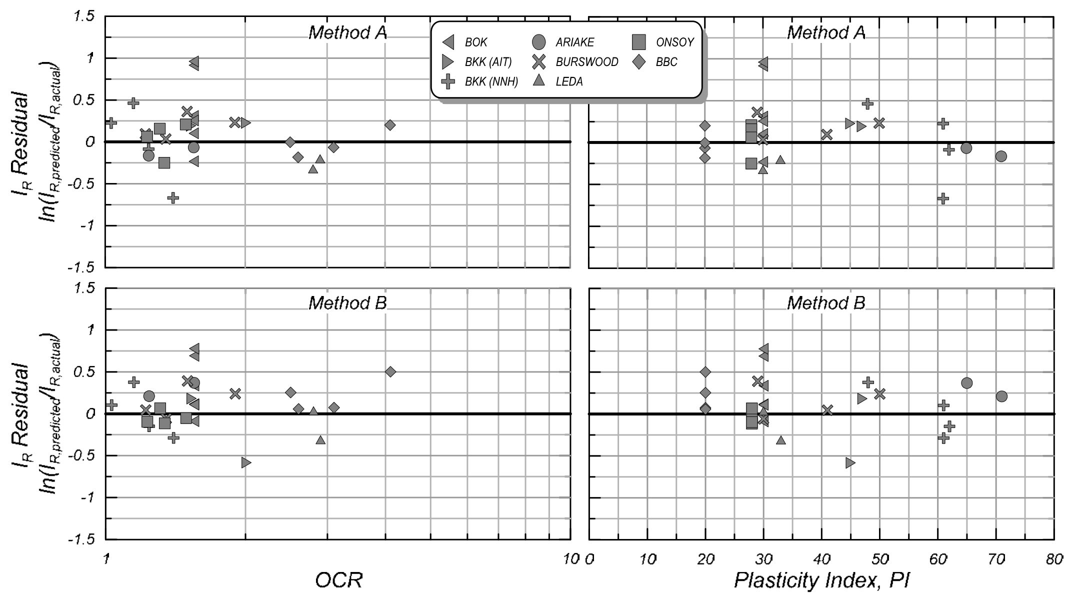

Figure 6. Residuals of the Empirical G_{50} / G_{max} Based Method A (top) and Method B (bottom) as a function of *OCR* (left) and *PI* (right)

7 CONCLUSIONS

A literature review was performed in order to determine the most appropriate shear modulus and undrained strength to determine I_R using the CPT (Broussard 2012). Researchers suggest the use of G_{50} is appropriate for I_R since it most likely represents an average response of the soil around an advancing cone. *CAUC* tests also appear to provide the most reasonable values for s_u mobilized around an advancing cone. An extensive database of lab tests from various clay sites was used to evaluate empirical trends and develop empirical relationships for estimating I_R. The trends in I_R as a function of *OCR* and *PI* were then used to develop two different approaches for estimating I_R (with and without the use of laboratory tests).

Several methods for predicting I_R were compared to measured values of I_R. On average the empirical methods (A and B) were shown to appropriately predict I_R. In order to verify the usefulness of CPT results, *OCR* correlations derived from the CPT were used to determine s_u using the SHANSEP framework. The results suggest that application of Method B (Eq. 6) using only seismic CPT data is useful if the soils follow the stress normalization of undrained strength concept presented in the SHANSEP methodology.

This paper has shown that the use of only an in-situ CPT can be sufficient for prediction I_R to a degree of uncertainty without the need for additional laboratory tests. *CAUC* tests on high quality samples are recommended for more precise estimations of I_R.

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