- $T_s = 0.0127$ Nm, motor stall torque
- $\omega_0 = (2\pi/60) \times 9200$ rad/s, motor no–load speed
- m, mass of load (kg)
- L, ramp length (m)
- θ, ramp angle (rad)
- r, spool radius (m)
- N, gearbox ratio (dimensionless)
- T, motor torque (Nm)
- ω, motor angular speed (rad/s)
- F, linear force (N)
- v, load velocity along ramp (m/s)
- s, distance along ramp (m)

Some fundamental relations among the above quantities:

$$T = T_s\left(1 - \frac{\omega}{\omega_0}\right), \quad rF = NT, \quad v = \frac{\omega r}{N}, \quad s = \int v\,\mathrm{d}t\,. \tag{1}$$

The equation of motion of the load up the ramp is

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = F - mg\sin\theta\,.$$

Substituting from the relations (1), this can be formulated as the differential equation

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{N^2T_s}{mr^2} - \frac{Ng\sin\theta}{r} - \frac{N^2T_s}{mr^2\omega_0}\,\omega\,.$$

for ω. By some re–arrangement, we obtain the more concise formulation

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{\omega_\infty - \omega}{\tau}\,, \tag{2}$$

where

$$\tau \;=\; \frac{mr^2\omega_0}{N^2 T_s} \qquad \text{and} \qquad \omega_\infty \;=\; \omega_0\left(1 - \frac{mgr\sin\theta}{NT_s}\right)$$

are the motor spin–up timescale and asymptotic (steady–state) speed. Note that, in order for ω_∞ to be positive, we must have

$$\frac{NT_s}{r} \;>\; mg\sin\theta\,,$$

i.e., the force in the line when the motor is at the stall torque must exceed the component of the load weight parallel to the ramp.

The solution to equation (2), subject to the initial condition $\omega = 0$ at $t = 0$, can be written as

$$\omega(t) \;=\; \omega_\infty[\,1 - \exp(-t/\tau)\,]\,. \tag{3}$$

In order for the run time predicted by steady–state analysis to be an accurate estimate of the actual run time, the timescale τ should be small compared to the predicted run time.

The exact run time, allowing for a non–negligible acceleration phase, can be computed as follows. From the third relation in (1) and (3), the time–dependent speed of the load up the ramp is

$$v(t) \;=\; \frac{r\omega_\infty}{N}[\,1 - \exp(-t/\tau)\,] \;=\; \frac{\mathrm{d}s}{\mathrm{d}t}\,.$$

Thus, if traversal of the ramp length L requires time Δt, we must have

$$\frac{r\omega_\infty}{N}\int_0^{\Delta t} 1 - \exp(-t/\tau)\;\mathrm{d}t \;=\; L\,,$$

or equivalently

$$(\Delta t/\tau) + \exp(-\Delta t/\tau) \;=\; \frac{NL}{r\omega_\infty\tau} + 1\,.$$

With $z = \Delta t/\tau$, the function $f(z) = z + \exp(-z)$ on the left satisfies $f(0) = 1$ and $f'(z) > 0$ for all $z \geq 0$, so there is a unique value of z for which the value of this function is equal to the expression on the right. The solution can be found by a simple iteration (e.g., Newton's method).