

ON THE PHYSICS OF DROUGHTS

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A SIMPLIFIED GENERAL CIRCULATION MODEL (GCM) AT MID-LATITUDES

In order to understand the dynamics of droughts, and the conditions under which they develop a simplified general circulation model (GCM) with coupled atmospheric-hydrologic components was developed for the strip of the Earth's northern hemisphere in the $30^{\circ} - 50^{\circ}$ N band.

We analyze the drought phenomena by means of the numerical simulations of the coupled atmospheric-hydrologic system by this simplified GCM.

In this simplified GCM only two atmospheric state variables, **the temperature T and the atmospheric water vapor content W** are simulated.

These state variables are intended to be the mean values through the depth of the atmosphere over mid-latitudes, specifically in the $30^\circ - 50^\circ$ N band.

They are averaged over the depth of troposphere, the lowest 10km or so of the atmosphere, and along the longitudinal direction.

Therefore, the only spatial coordinate left is the west-east direction, denoted by “ x ”.

The state of the hydrologic system is represented by only two variables:
water storage GW and ground temperature T_g .

Ocean temperature is a specified function of the season only.

The coupling between the atmosphere and the Earth's surface occurs through exchanges of thermal energy (radiation, sensible and latent heat fluxes) and of water (ET and rainfall), which are parameterized functions of the selected state variables of the atmospheric and hydrologic components of the Earth system.

Hence, the simplified GCM which has only 4 state variables, is expected **to qualitatively represent** the effect of a series of meteorological events in terms of time-averaged (climate-scale) quantities.

THERMAL ENERGY BALANCE IN THE ATMOSPHERE (HEAT TRANSPORT EQUATION)

Integration of the thermodynamic energy equation through the depth z_0 of the troposphere yields the heat transport equation:

$$c_p M \frac{dT}{dt} = c_p M \left(\frac{\partial T}{\partial t} + \mathbf{V}_T \cdot \nabla T \right) = -\omega Z_0 + \frac{dQ}{dt}$$

where T and ω are respectively the average temperature ($^{\circ}\text{K}$) and pressure velocity (Pa Sec^{-1}) in the troposphere, and dQ/dt (Wm^{-2}) is the external diabatic heating rate. C_p is the specific heat of air at constant pressure ($C_p = 1004 \text{ J } ^{\circ}\text{K kg}^{-1}$) and M is the air mass per unit area through the depth of the troposphere ($\sim 6000\text{kg}$).

In order to solve this equation one may expand the temperature field as :

$$T(x, t) = \sum_{i=1}^n T_i(t) \phi_i(x - x_i(t))$$

where $T_i(t)$, $i=1, \dots, n$ are the expansion coefficients and $\phi_i(x)$, $i=1, \dots, n$ are the basis functions, to be defined later.

$x_i(t) = x_{oi} + c_i t$, $i=1, \dots, n$ move the temperature waves eastward at the wave speeds c_i .

ATMOSPHERIC WATER BALANCE

(ATMOSPHERIC WATER VAPOR TRANSPORT)

Conservation of water vapor through the depth of the troposphere may be expressed as:

$$\frac{dW}{dt} = \frac{\partial W}{\partial t} + V \cdot \nabla W = ET - C$$

where; W is the mass of water vapor in a vertical column of the troposphere with unit base (kg/m^2)

$V \cdot \nabla W$ is the advection of water vapor ($\text{kgm}^{-2}/\text{sec}$)

ET is the rate of evapotranspiration input from the ground/ocean surface ($\text{kgm}^{-2}/\text{sec}$)

C is the precipitation rate ($\text{kgm}^{-2}/\text{sec}$)

To model advection of water vapor, expand the water vapor content in the form

$$W(x, t) = \sum_{i=1}^n W_i(t) \phi_i[x - \xi_i(t)]$$

where $W_i(t)$, $i=1, \dots, n$ are the expansion coefficients of W

$\phi_i[x]$, $i=1, \dots, n$ are the basis functions

$\xi_i(t) = \xi_{0i} + U_i t$, $i=1, \dots, n$ move the water vapor eastward at wind speed U_i

CHOICE OF BASIS FUNCTIONS:

In the simple GCM the basis functions are represented by a constant and half-cycle sine waves with varying periods (wavelengths).

The constant represents the average conditions of temperature and water vapor content, whereas the sine waves model the weather disturbances. For their wavelengths on the synoptic scale a range from 2000 to 10000km is taken.

From the above representations one may see that the basis functions are moved eastward at different celerities. In the simple GCM the natural behavior of the atmosphere, where temperature and vapor waves are moved eastward in the mid-latitudes primarily due to advection by mean winds, is simulated by shifting the basis functions by varying celerities.

For temperature waves, assuming a barotropic behavior of the atmosphere (ie. neglecting the influence of fronts on the dynamics of the atmosphere), Rossby wave theory applies (Rossby et al. 1939; Holton, 1979).

The relationship between celerities c_i and half-wavelengths L_i is shown in the below table.

Celerities c_i of temperature (T) waves, and advection speed U_i of water vapor content (W) waves associated with basis functions of half wavelength L_i , as used in the simplified GCM

ϕ_i	2	3	4	5	6	7	8	9	10	11	12	13	14
$L_i(\text{km} \times 1000)$	5.0	4.5	4.0	3.5	3.25	3.00	2.75	2.50	2.25	2.00	1.75	1.50	1.25
$c_i(\text{km h}^{-1})$	0.3	5.4	9.9	13.9	15.7	17.4	18.9	20.3	21.6	22.7	23.7	24.6	25.3
$U_i(\text{km h}^{-1})$	20.0	18.0	16.0	14.0	13.0	12.0	11.0	10.0	9.0	8.0	7.0	6.0	5.0

The relation between c_i and L_i is based upon Rossby wave theory, but numerical values have been adjusted to avoid having westward-moving temperature waves.

HDROLOGIC WATER BALANCE

The drought intensity is quantified by the water stored in the modeling domain, GW.

GW denotes the water below or above a land surface of unit area. In this simple GCM it is assumed that a simple water balance governs the storage of water:

$$\frac{dGW}{dt} = -k \cdot GW - ET + C$$

In this equation the first term on the rhs represents the water losses due to water outflow (river runoff and subsurface flow),

ET is the evapotranspiration rate,

C is the precipitation rate.

All terms are per unit area.

k represents the recession constant for the land basin storage, taken as a linear storage. A value of $k = 1.4 \times 10^{-8}/\text{sec}$ gives the correct order of magnitude for the mean runoff rate when compared to the data measured by Levitovich and Ovtchinnikov (1964).

EVAPOTRANSPIRATION:

Evaporation from ocean surfaces:

$$ET = c_{ET}(w_{ws} - w_0)$$

C_{ET} = ratio of air density to aerodynamic resistance ($0.04 \text{ kg m}^{-2}/\text{sec}$)

w_{ws} = saturation mixing ratio at the water surface

w_0 = actual mixing ratio at height $z = 2\text{m}$ above water surface (kg/kg)

The saturation mixing ratio may be estimated from observations at 1000mb (NWS), by using a best-fit expression:

$$w_{ws} = 10^{-4}(39 + 2.7T_{oc} + 0.068T_{oc}^2 + 0.0032T_{oc}^3)$$

where T_{oc} is the ocean temperature.

Over land surfaces:

(Thorntwaite and Matter, 1955; Budyko, 1956)

$$ET = c_{ET} \frac{GW - GW_{\min}}{GW_{\max} - GW_{\min}} (w_{gs} - w_0)$$

where

$GW_{\max}$ = maximum storage of water to achieve potential ET

$GW_{\min}$ = GW value when ET ceases

W_{gs} = saturation mixing ratio calculated by the previous equation in terms of ground temperature T_{oc}

c_{ET} = 1.5 times the c_{ET} for oceans since large sensible heat fluxes can develop due to free convection over land surfaces.

SENSIBLE HEAT FLUX (W/m²)

$$H_g = 1.43c_{ET} \bullet (T_g - T_b)$$

where

$$T_b = T + 0.5\gamma(z_t - z_b)$$

PRECIPITATION

Precipitation is assumed to occur when supersaturation conditions are attained in the atmosphere. Accordingly, the large scale precipitation is modeled by means of the Clausius-Clapeyron equation (Holton, 1979)

$$C = \frac{w - w_s}{1 + L^2 w_s / c_p R_v T^2} \bullet \frac{m}{\Delta t}$$

C = precipitation rate (kg/m²-sec)

w = average mixing ratio of water vapor in the troposphere (kg/kg)

w_s = average saturation mixing ratio of water vapor in the troposphere (kg/kg)

m = fractional weight of water vapor in an atmospheric column at which precipitation starts
(2000 kg/m²)

R_v = gas constant for moist air (461 J/kg/K)

L = latent heat of condensation (2.5X10⁶ J kg⁻¹)

Δt = computational time interval

LAND SURFACE TEMPERATURE

The land surface temperature is determined by an energy balance:

$$z_g c_g \frac{dT_g}{dt} = R_n - H_g - L \cdot ET$$

T_g = land surface temperature ($^{\circ}\text{K}$)

z_g = effective depth of soil layer (taken as 0.5m)

c_g = soil thermal capacity ($2.5 \times 10^5 \text{ J/m}^3\text{-}^{\circ}\text{K}$)

R_n = net radiative flux at soil surface (W/m^2)

H_g = sensible heat flux at soil surface (W/m^2)

L = latent heat of condensation

HEATING/COOLING RATE IN THE HEAT TRANSPORT EQUATION

$$\frac{dQ}{dt} = S_R + G_R - L_R + H_g + LC$$

S_r = shortwave radiation absorbed in the atmosphere (W/m^2)

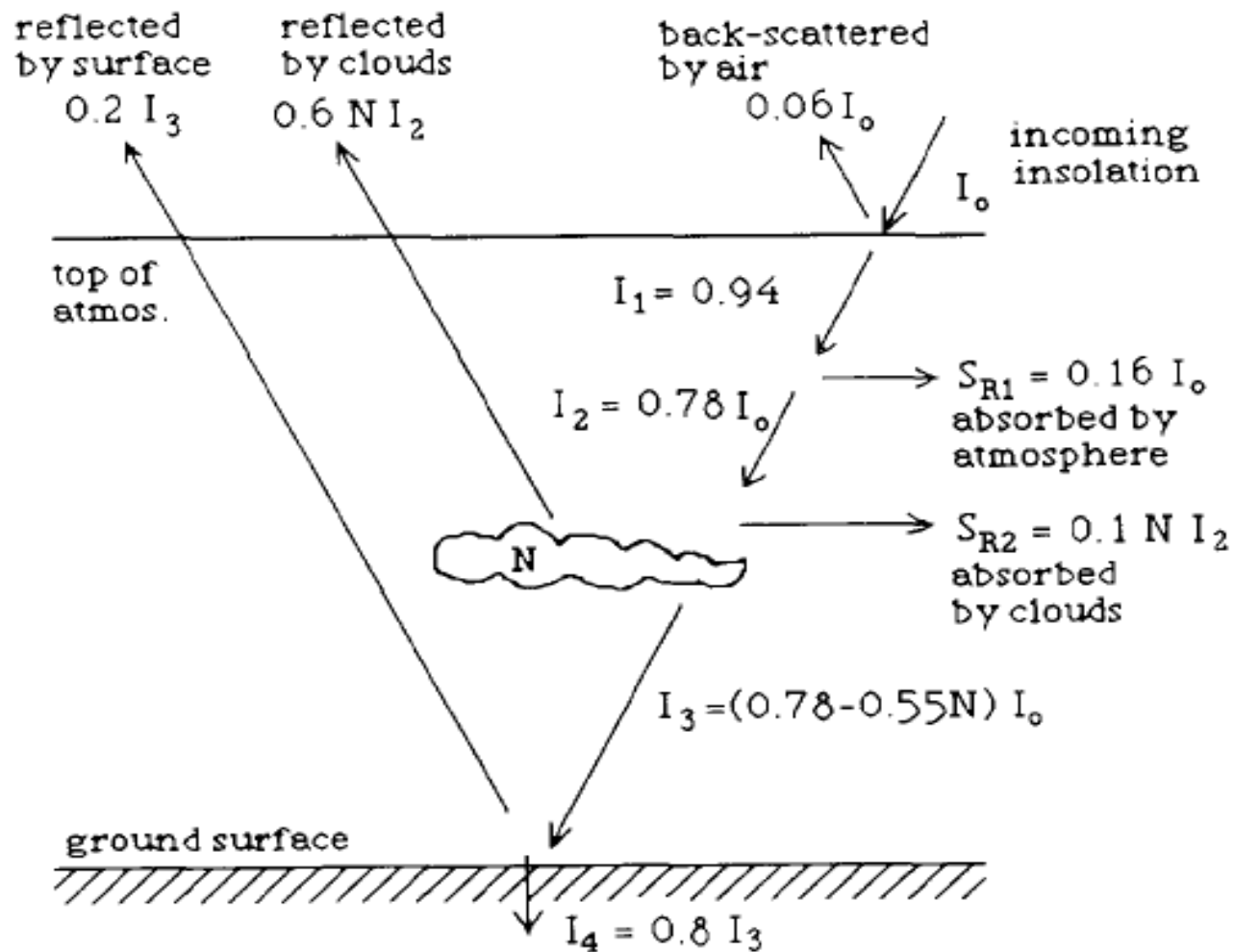
G_r = atmospheric absorption of longwave radiation that is emitted by ground surface (W/m^2)

L_r = longwave radiation emitted by the atmosphere (W/m^2)

H_g = sensible heat flux absorbed in the atmosphere (W/m^2)

L = latent heat for condensation ($2.5 \times 10^6 \text{ J kg}^{-1}$)

C = precipitation rate ($\text{kg sec}^{-1} \text{ m}^{-2}$)



Partitioning of incoming solar radiation. Numerical values are adopted from Eagleson (1970), Corby et al. (1977), and Wallace and Hobbs (1977)

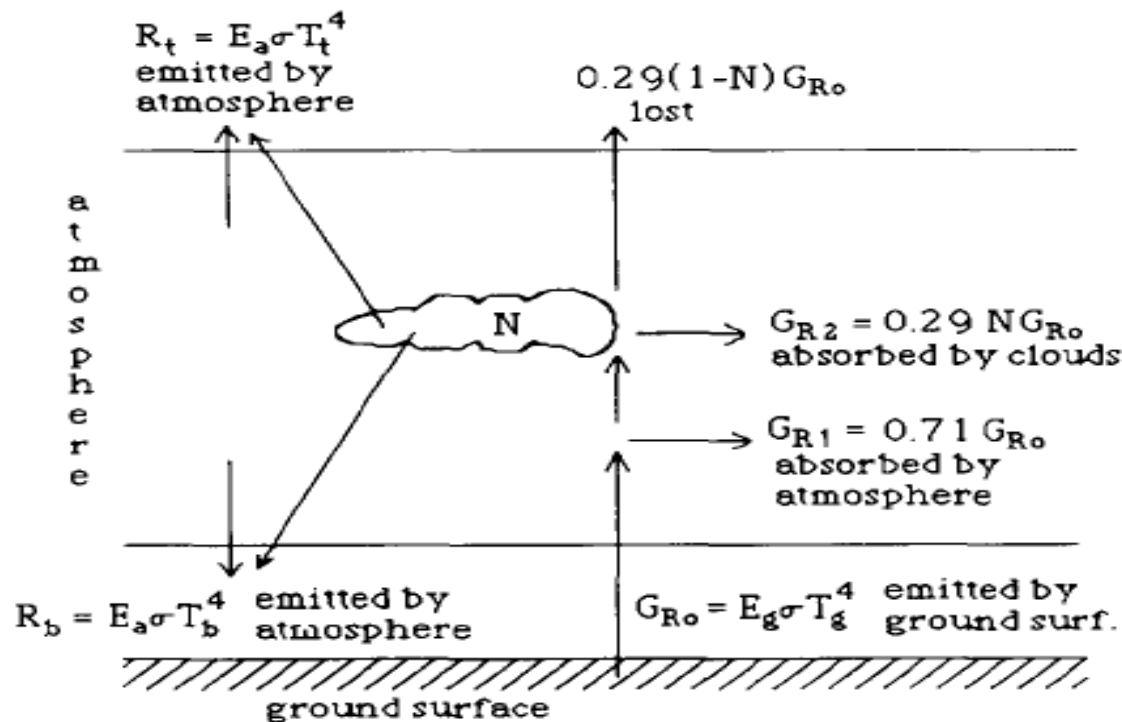
Longwave radiation balance:

For the longwave radiation balance, a simplified picture is considered for the simplified GCM. As the state variable is defined through a single value of average temperature, the vertical thermal structure is defined by an average lapse-rate of temperature $\sim 6\text{-}7^\circ\text{C/km}$ (Wallace and Hobbs, 1977). Hence, the temperature at the bottom of the atmosphere:

$$T_b = T + 21^\circ\text{C}$$

and the temperature at the top of the atmosphere:

$$T_t = T - 21^\circ\text{C}$$



Numerical values were adopted from Wallace and Hobbs (1977) and Eagleson (1970)

COMPARISON WITH CLIMATOLOGICAL AVERAGES:

The simplified GCM was run for 300 years to obtain data in terms of monthly averages at 1000km grids for temperature, mixing ratio, precipitation, etc. These values were then compared against observed climatological averages in order to slightly calibrate the simplified GCM to match the observed averages.

Comparison of GCM climatological averages with actual data taken from Sellers (1965, 1969) and Budyko (1982)

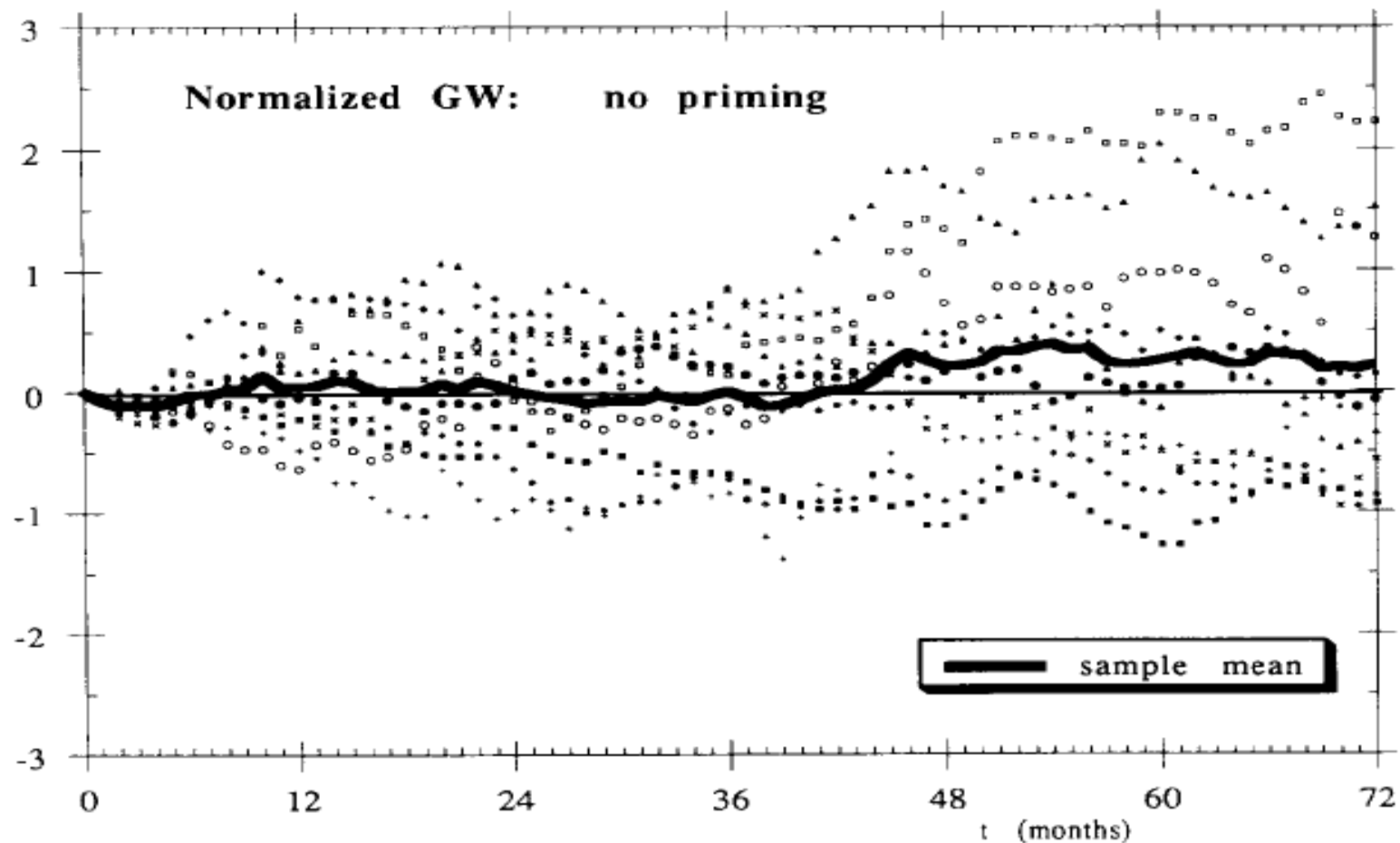
Quantity	GCM average value	Actual data (30–50°N)
Precipitable water (kg m^{-2})	16	17
Air temperature at surface ($^{\circ}\text{C}$)	13.2	13
Storage of water (GW) (kg m^{-2})	1050	–
Soil temperature ($^{\circ}\text{C}$)	13.8	–
Cloudiness over land	0.49	0.50
Cloudiness over oceans	0.37	–
Evaporative heat flux (LE_T) over land (W m^{-2})	35	32
Evaporative heat flux (LE_T) over ocean (W m^{-2})	103	93
Overall evapotranspiration (mm year^{-1})	876	822
Overall precipitation (mm year^{-1})	918	890
Net radiation over land (W m^{-2})	60	70
Net radiation over ocean (W m^{-2})	93	90
Sensible heat flux (H_g) over land (W m^{-2})	25	19
Sensible heat flux (H_g) over ocean (W m^{-2})	54	39

All of the simplified GCM simulations start from the initial conditions (lcs) that correspond to the average conditions on 31 December at all geographical locations, except the Western USA region where a stationary wave of amplitude T_p ($^{\circ}\text{C}$) is hypothesized to be present for a duration of d_p months. The longest temperature wave must be used since stationary disturbances are observed only for very long wavelengths.

Eventhough the initial conditions for the atmospheric temperature T and water vapor content W are fixed, one still has the freedom to vary the initial positions of the T and W waves which are initially of zero amplitude. Therefore, the simulations can be repeated starting with the same lcs except varying the locations of the null waves.

By placing a T or a W wave at some region it is assumed that there is the potential for the growth of one of these waves in that region. Whether the wave will develop or not depends on the state of the system.

It must be shown that the initial conditions that are used for all the simplified GCM simulations do not induce by themselves an average trend in atmospheric and/or hydrologic behaviors. To test this issue the simplified GCM was run for 10 different initial positions of the T and W waves. The below figure shows the results of individual runs and their ensemble average in terms of the surface water storage GW (monthly averages) for the 1000km grid located over the Western USA in nondimensional, standardized form.



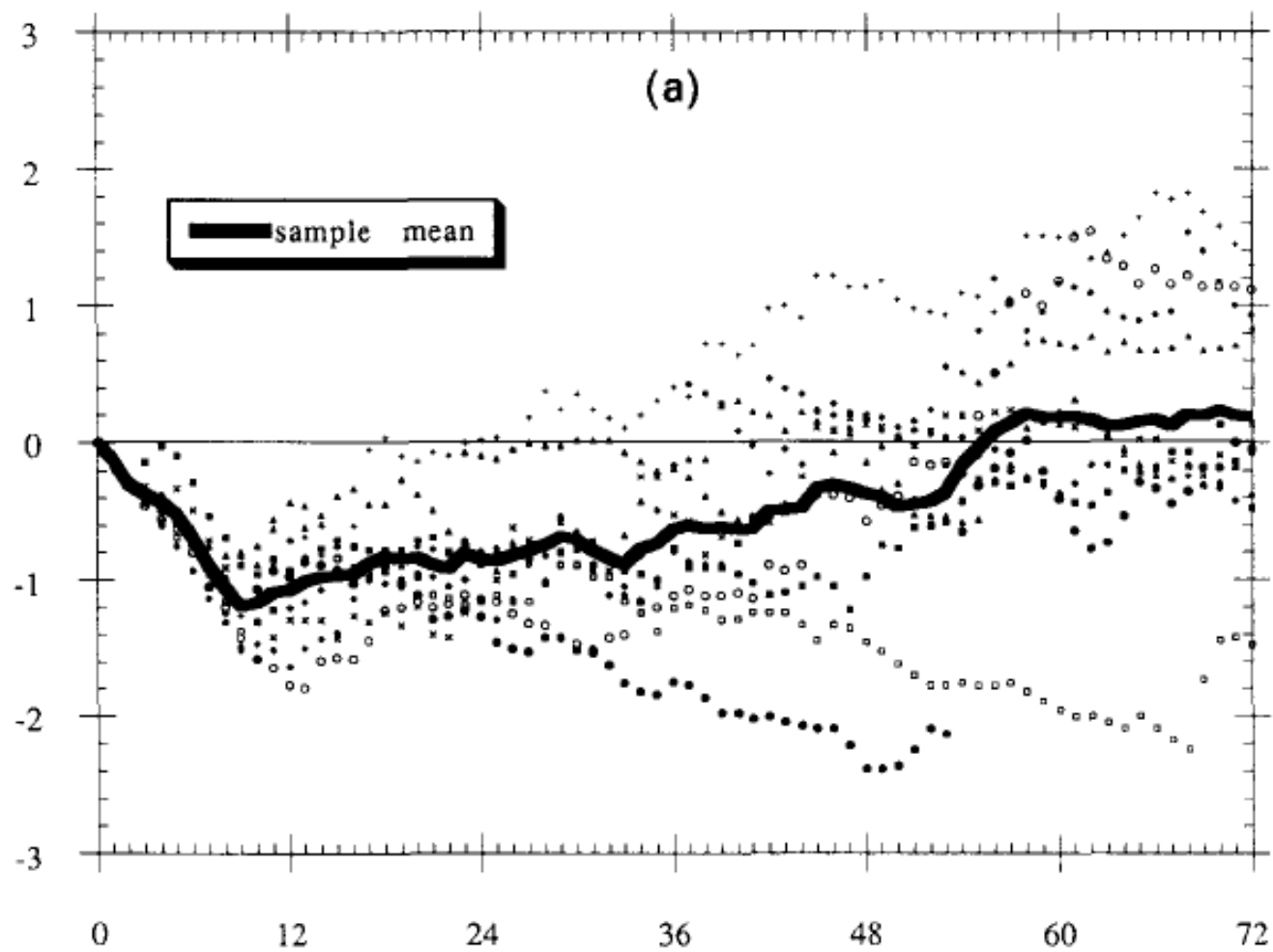
THE POSITIVE FEEDBACK MECHANISM FOR THE INITIATION OF DROUGHTS

By imposing a stationary temperature wave on the initial conditions for some period of time over a grid, drought conditions may be produced. Accordingly, 10 simplified GCM simulations over the Western USA grid for a stationary heat wave of amplitude $T_p = 1^\circ\text{C}$, for a duration of $d_p = 1$ month that was superimposed on the initial conditions over that grid, are shown in the below figures.

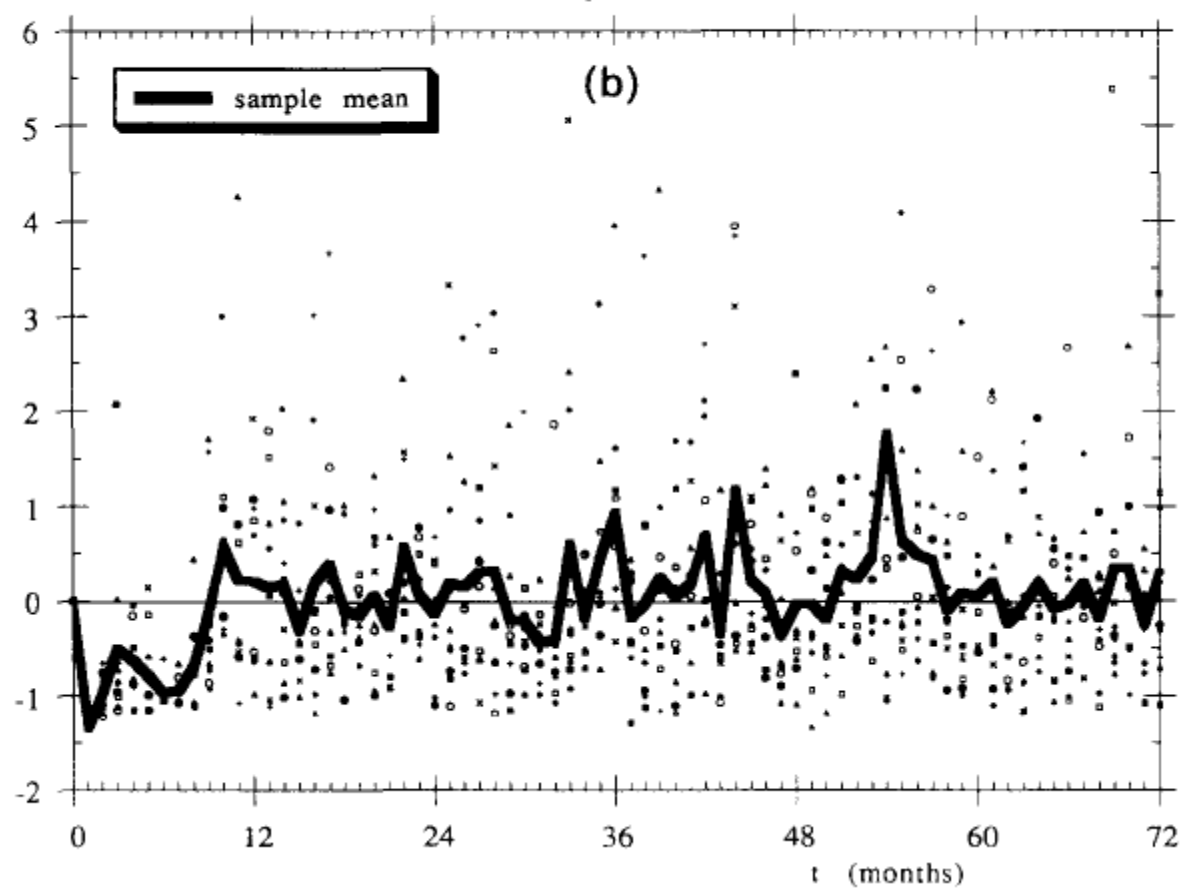
In these figures, while the individual realizations of the GCM simulations of the Earth's nonlinear system look chaotic, their ensemble averages show some clear trends.

For the water storage GW, while the anomalous stationary heat wave is introduced only for one month over the Western USA grid, the ensemble average GW keeps on decreasing for almost one year, mainly due to a significant reduction in rainfall. Almost 5 years pass before the ensemble average GW reclaims its initial state.

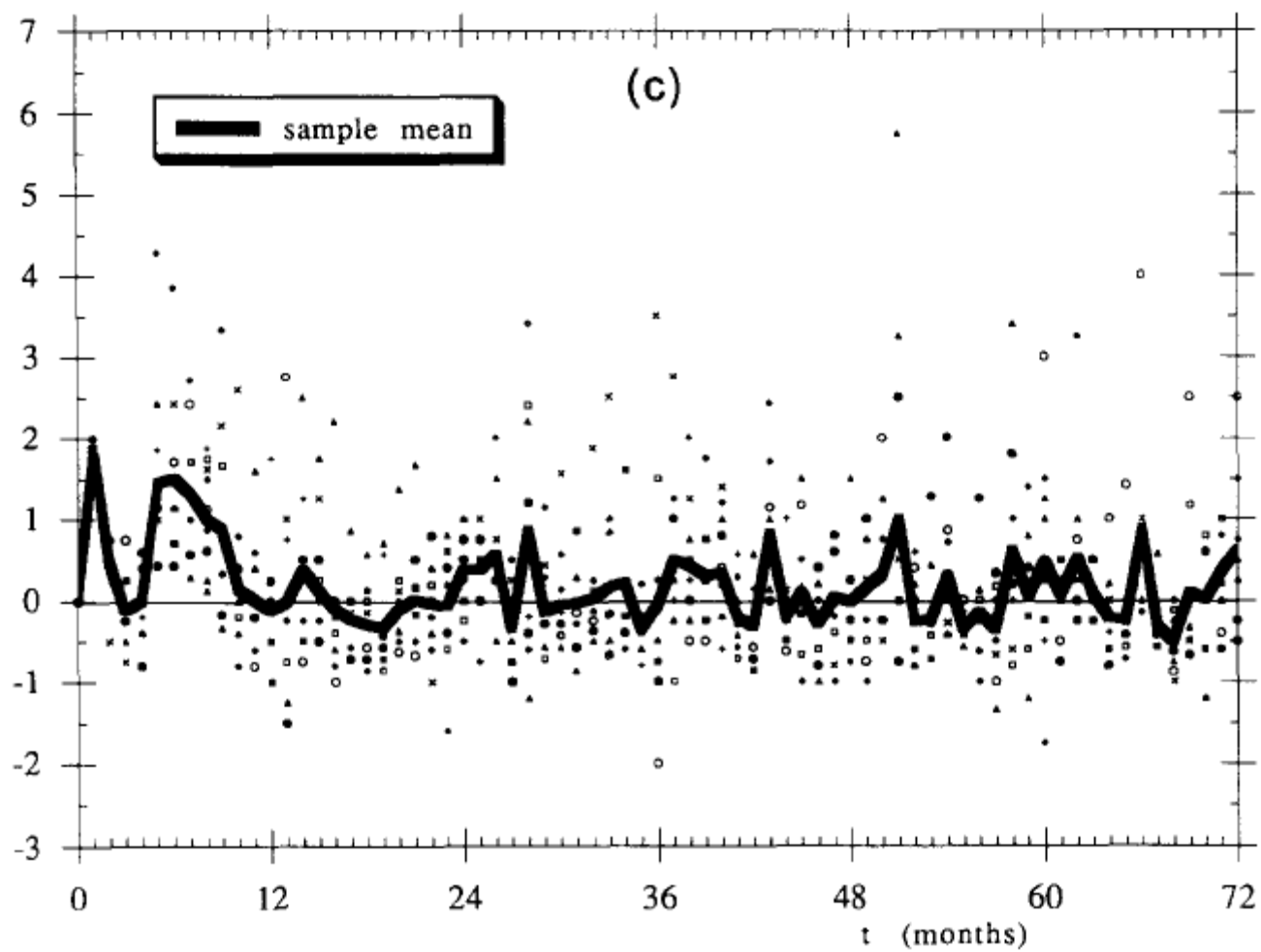
Normalized GW: $T_p = 1^\circ\text{C}$, $d_p = 1$ month



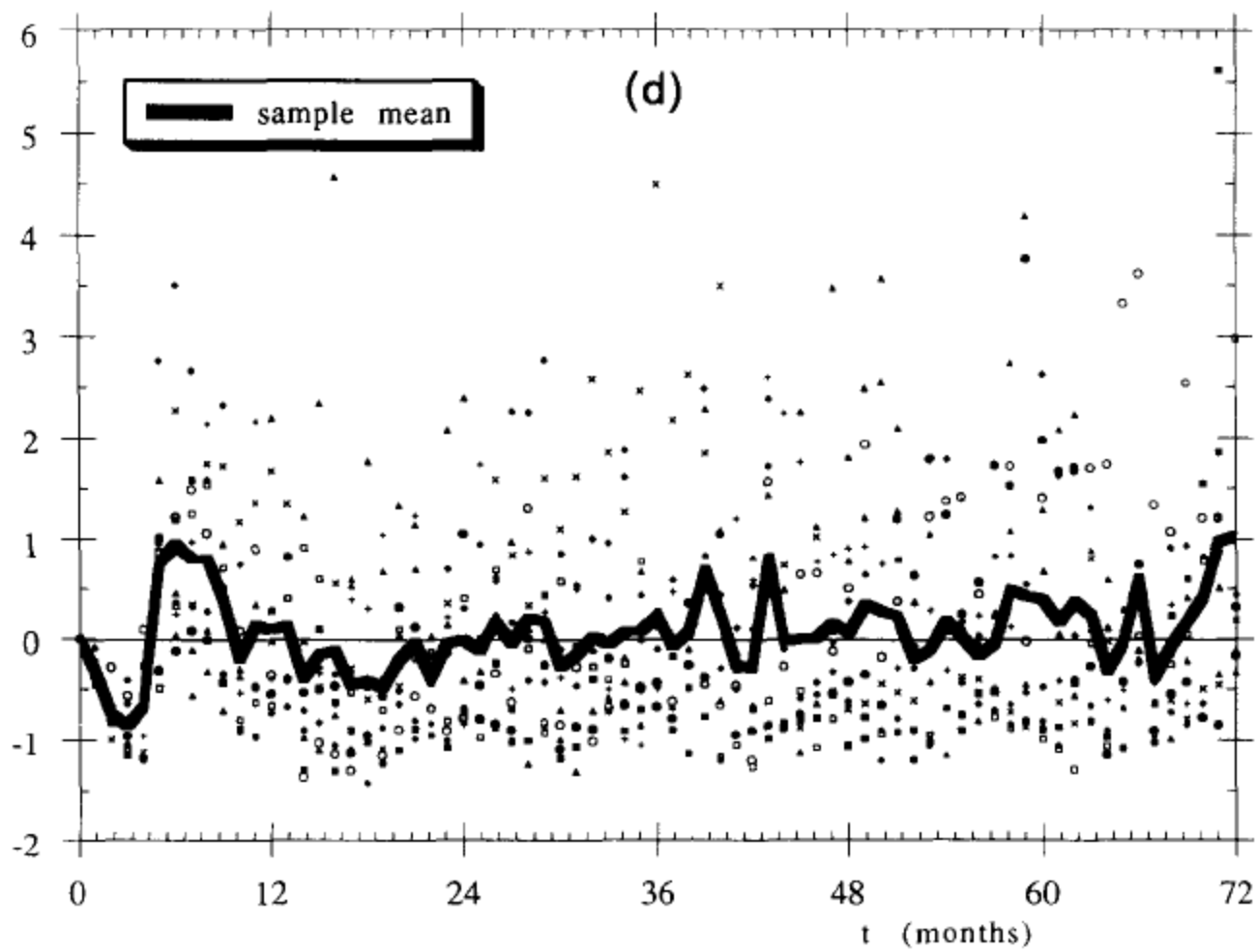
Normalized rain: $T_p = 1\text{ }^{\circ}\text{C}$, $d_p = 1\text{ month}$

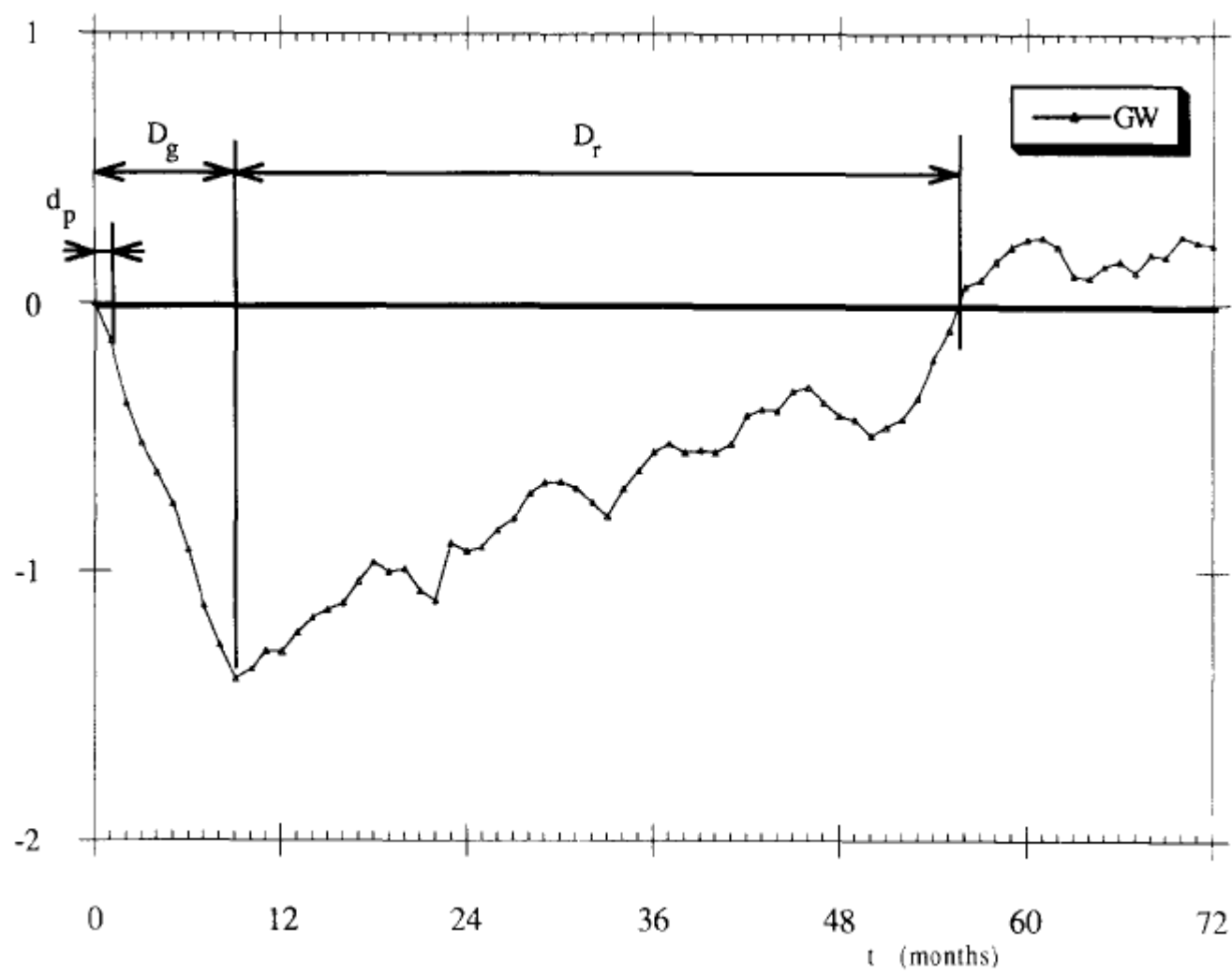


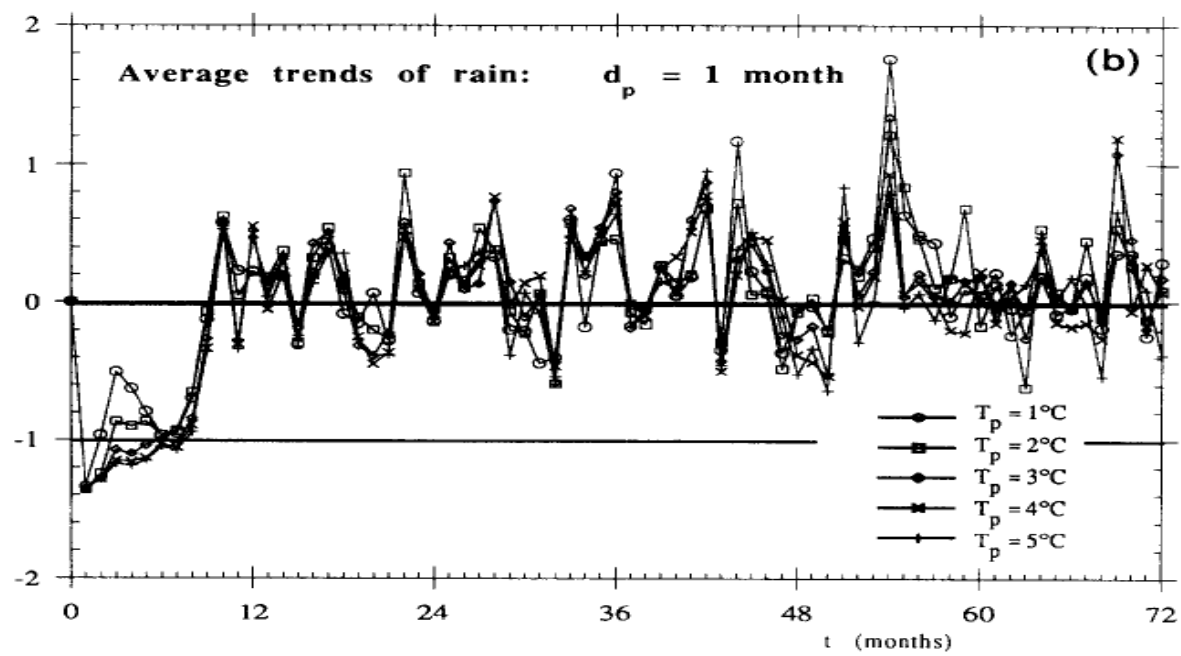
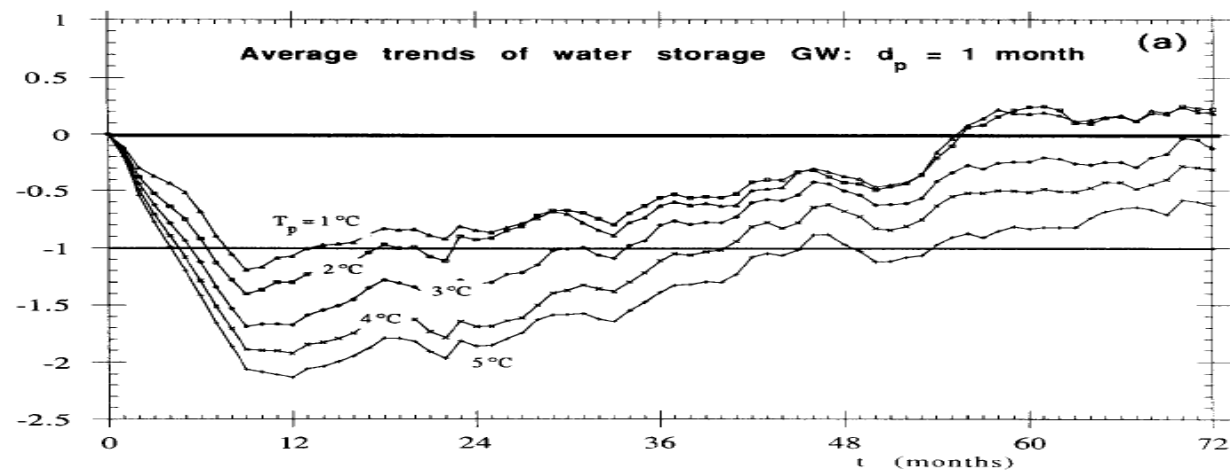
Normalized T_{soil} : $T_p = 1^\circ\text{C}$, $d_p = 1$ month

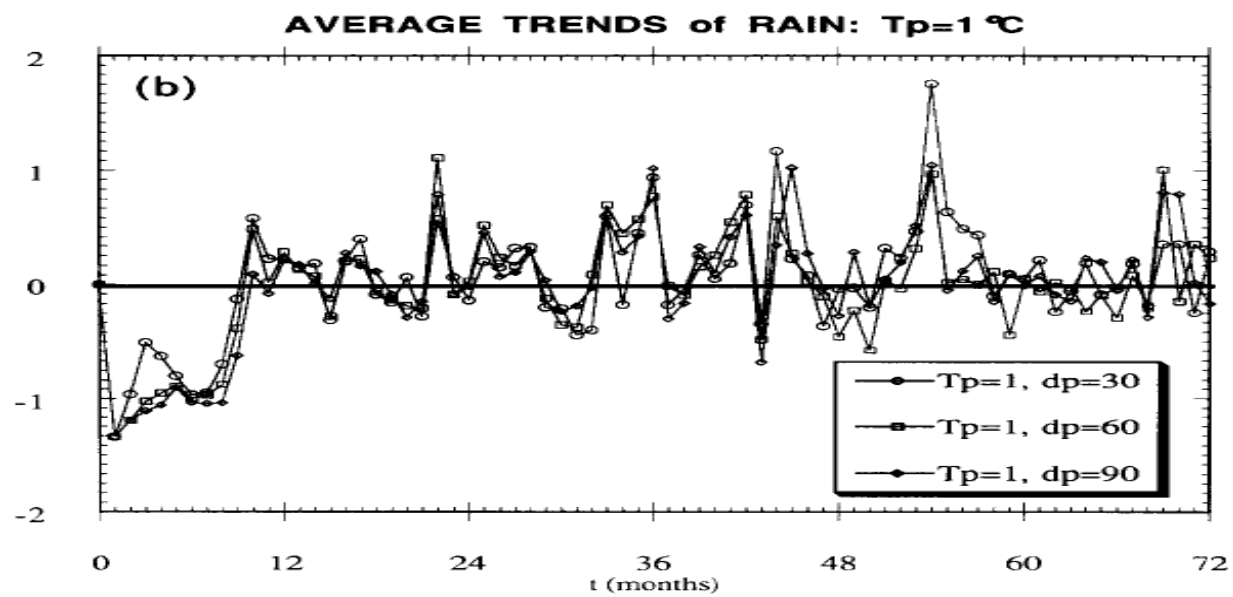
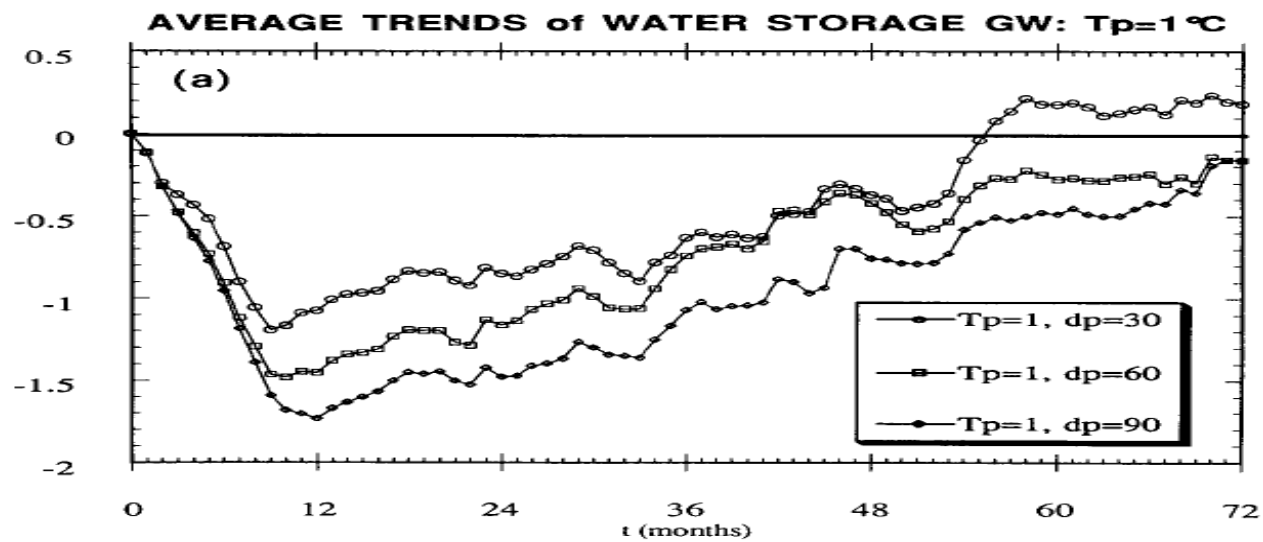


Normalized ET: $T_p = 1\text{ }^{\circ}\text{C}$, $d_p = 1\text{ month}$









CONCLUSIONS:

From the simulations by a simplified GCM it is seen in the ensemble average sense that droughts over a region, if defined in terms of the combined surface-subsurface water storage in that region, are not only a function of the deficit in precipitation but may also be due to other geophysical factors such as atmospheric temperature, humidity, cloudiness and radiation, and ground temperature and evapotranspiration.

Short period anomalous heat wave forcings over a region, can produce long-lasting drought conditions at the meso-climatic time scale in the ensemble average sense.

On the ensemble average sense, the simulations of the earth's geophysical system by a simplified GCM have shown that the drought conditions are nonlinear in the sense that the drought-causing mechanisms feed on themselves. Imposing an anomalous temperature wave over a region for a relatively short time (~ 1 month) induces geophysical conditions in that region which reduce the precipitation for a much longer period of time ($\sim$ one year).

The drought recovery period which is necessary to return to the climatological average conditions for the water storage, is proportional to the peak severity of the drought that is reached during the growth period.

Due to the inherent nonlinearity of the Earth's system, single realizations of the GCM simulations in this study do not necessarily follow the above observations which are valid only in the ensemble average sense.