

# LEAP Projects: Concept and Challenges

M.T. Manzari,

*The George Washington University, Washington DC, USA*

B. L. Kutter,

*University of California, Davis, California, USA*

M. Zeghal,

*Rensselaer Polytechnic Institute, Troy, New York, USA*

S. Iai, T. Tobita,

*Disaster Prevention Research Institute, Kyoto University, Kyoto, Japan*

S. P. G. Madabhushi, S. K. Haigh,

*Cambridge University, Cambridge, UK*

L. Mejia,

*URS Corp., Oakland, CA, USA*

D. A. Gutierrez, R. J. Armstrong

*Division of Safety of Dams, Sacramento, CA, USA*

M. K. Sharp,

*US Army Corps of Engineers, Mississippi, USA*

Y. M. Chen, Y.G. Zhou

*Zhejiang University, Hangzhou, Zhejiang, China*

**ABSTRACT:** The Liquefaction Experiment and Analysis Project (LEAP), an international research collaboration among researchers from the US, UK, Japan, China and Taiwan, is a validation campaign to assess the capabilities of existing numerical/constitutive models for liquefaction analysis by using laboratory experiments and centrifuge tests. The nature, goals, and scope of the project are presented in this paper. The goals of the planning phase of the project that is currently ongoing in the US are briefly discussed. The main components of the validation campaign and their corresponding challenges are outlined.

## 1 BACKGROUND

### 1.1 *The need for a new validation campaign*

Liquefaction-induced permanent deformations and failure in geo-structures such as retaining structures, soil slopes, and earth embankments remain a major concern to the geotechnical engineering community. Following large earthquakes, recorded data, field investigations, and various case studies have often been used to understand the mechanisms of failure and to establish a link to key features of soil stress-strain-strength behavior. Furthermore, intensive efforts have been undertaken by researchers towards the development of constitutive and numerical modeling tools capable of predicting cyclic and permanent deformations of liquefaction prone soils (e.g., Zienkiewicz et al., 1998; Elgamal et al., 2003; Manzari and Dafalias, 1997; Dafalias and Manzari, 2004; Jeremic, et al., 2008). Except for occasional efforts such as that by EPRI (1993), thorough assessment and validation of these computational tools has been limited.

In a recent paper, Perlea and Beatty (2010) explain how advanced numerical methods are now being used by the US Army Corps of Engineers to evaluate the effects of seismic loading and liquefaction on dams. They describe the use of three different codes including FLAC (with a hypoplasticity model, Wang et al., 2006; with UBCSand model, Byrne et al., 2003; and with an empirical pore pressure generation model, Dawson et al., 2001), DYNFLOW (Prevost, 1983), and TARA (Finn et al., 1986) in the assessment of the seismic response of a number of earth dams. The extensive computational work reported in Perlea and Beatty (2010) clearly demonstrates the continuing need of practicing engineers for validation and assessment of modern numerical tools that are now available for geotechnical analysis.

In the early 1990's, over 20 teams of numerical modelers participated in an elaborate prediction exercise that was meant to Verify Liquefaction Analyses by Centrifuge Studies (VELACS) (Arulanandan and Scott 1993-1994). Centrifuge tests were conducted and duplicated at different centrifuge facili-

ties in the US (UCD, RPI, University of Colorado at Boulder, and Princeton University) and Cambridge University in the UK. A large number of “Class A” (i.e., true prediction of an event made prior to the event) numerical simulations of these centrifuge tests were submitted and compared at a symposium. The two key lessons learned from this exercise were that: (1) none of the numerical techniques available at that time were reliable for producing high quality predictions of liquefaction problems, and (2) there was significant variability in many of the centrifuge test results.

Over the past few decades, the geotechnical engineering community has seen remarkable advances in experimental and computational simulation capabilities. Experimental research using increasingly reliable element scale laboratory tests, in-situ tests, and centrifuge experiments have provided the community with significantly improved understanding of the response of geosystems to earthquake loading.

In the same vein as the VELACS project, a recent exercise was conducted in Italy, on predicting the tunnel-soil interaction using numerical procedures that are matched to centrifuge test data. This project titled Round Robin Tunnel Tests (RRTT) has involved seven different numerical modeling teams that were involved in predicting the centrifuge test results in terms of tunnel lining forces and bending moments amongst other parameters, Bilotta et al (2014).

The tremendous advances in computational power and computational methods have provided an unprecedented opportunity for the analysis of very large geo-structural systems using sophisticated constitutive and numerical modeling techniques. Compared to 25 years ago, there are far better computational and numerical modeling techniques available for the analysis of soil liquefaction and its consequences. In the realm of constitutive modeling, there are several well-established constitutive models for saturated granular soils (Elgamal, et al., 2003; Ling and Yang, 2006; Manzari and Dafalias, 1997; Cubrinovski and Ishihara, 1998; Dafalias and Manzari, 2004; Taiebat, 2009). Moreover, several commercial finite element/finite difference codes (e.g., FLAC) provide nonlinear fully-coupled effective stress capabilities for analysis of geotechnical structures involving liquefiable soils. New advances in meshfree (Manzari and Regueiro, 2005), finite element analysis (Regueiro and Borja, 2001; Manzari, 2004; Manzari and Yonten, 2010, 2011-a, 2011-b) and discrete element techniques (Zeghal and El Shamy, 2004) have provided the community with

powerful tools to model liquefaction as well as post-failure response of geotechnical structures.

All these advanced computational tools still need to be assessed and validated against high-fidelity experiments. Without validation, the profession will remain rightfully skeptical and reluctant to adopt such tools. Given the significant advances in numerical modeling over the past twenty five years, it is time for a reassessment of the reliability of modern numerical modeling techniques in analysis of geotechnical engineering problems involving liquefaction.

## 1.2 Development of LEAP

The Liquefaction Experiments and Analysis Project (LEAP) is an international effort to produce a set of high quality centrifuge test data that can be used for validation of existing numerical simulation procedures for liquefaction analysis in a class-A prediction exercise. Evaluation of a wide range of analysis procedures requires a wide range of experiments and analysts. International collaboration widens the scope, gravitas, and impact of the findings. LEAP is an ongoing collaboration that encompasses the writers and their research groups and a team of researchers from the UK led by Cambridge University, the geotechnical earthquake engineering research group at Kyoto University, the geotechnical engineering group at National Taiwan University, and researchers from Zhejiang University in China.

The birth of the LEAP collaboration may have been in November 2011 when the writers submitted a NEESR proposal that included significant international collaboration. The writers had shared their proposal with a team of Japanese and a team of UK researchers, and in return they committed to submit parallel proposals to their respective national funding agencies.

Fortunately, the Japanese proposal, led by Susumu Iai from Kyoto University, was successfully funded and they began performing experiments and analyses at multiple institutions in Japan in 2012. Manzari, Kutter, and Madabhushi were invited to attend a meeting in Kyoto on January 30-31 2013 that included a first phase prediction exercise.

On the first day of the meeting simulations of experiments from three different centrifuge facilities were compared. The experiments modeled liquefying level and sloping ground. Simulations included use of numerical simulation codes such as FLIP ROSE, FLIP TULIP, LIQCA (Iai et al., 1992, 2011), and the FE simulation software developed by Manzari. These comparisons reminded us how im-

portant it is to carefully model boundary conditions in both the centrifuge and the analyses, and proved once again that nonlinear finite element simulations of liquefaction can be prone to error, and experiments of liquefaction can produce variable results when using different equipment.

On the second day of the Kyoto meeting a smaller group of international collaborators (from Taiwan, Japan, UK and the US) met to discuss how international efforts could be synergistically coordinated. The group came up with the name *LEAP* (Liquefaction Experiments and Analysis Project), and an overall objective/goal: To evaluate the capability of a wide range of analysis procedures to accurately predict the response (especially deformations) of geotechnical structures including effects of liquefaction.

Details of the project are further discussed in the next sections.

## 2 NATURE AND GOALS OF THE PROJECT

### 2.1 *Kyoto Document*

A document describing the nature and goals of the collaboration was produced following the Kyoto meeting. The Kyoto document states that at least 3 LEAP “projects”, each involving a focused set of experiments and simulations are presently envisioned. For example: (1) Level ground and sloping ground, (2) retaining structure (sheet pile or seawall), (3) embankment and/or embankment dams, and (4) additional focused sets of experiments depending on the sub disciplines of the funding agencies (tunnels, ports, transportation infrastructure, levees, etc.).

It is envisioned that each project under LEAP could proceed in parallel but out of phase with the other projects. Each LEAP project would be led and hosted by one national LEAP team, but international participants will be invited and expected to participate. The leader of each LEAP project will become a member of the international steering committee, which provides a mechanism for coordination.

Protocols for specifying experiments, simulations, material properties and constitutive model calibrations, as well as results of experiments and simulations would be shared amongst all the LEAP projects. A core group for each project will make the specifications for the experiment and simulations for each blind prediction exercise. Broader participation by other physical and/or numerical simulation teams will then be invited. Each LEAP project would culminate in an international prediction sym-

posium. The plan has the following benefits and advantages:

- LEAP will involve different problem-focused projects that will each attract different agencies and sub disciplines.
- Lessons learnt in the earlier projects can benefit planners of the later LEAP projects.
- Class A prediction data developed in one LEAP project can be used as Class C (i.e., predictor validates a model by comparing results to known data) prediction data in later LEAP projects
- Material properties and constitutive law calibrations developed in one LEAP project can be used in later LEAP projects
- Making LEAP projects asynchronous allows different international teams to have different levels of activity at different times depending on the ebb and flow of their team’s funding.
- Feedback from physical modelers to numerical analysts and vice versa in successive LEAP projects will lead to a gradual improvement in the process of experimental validation of numerical procedures and best practices for experiments and analysis.

### 2.2 *Planning Phase of LEAP in the US*

The writers are currently involved in a research project that lays the ground for a LEAP in the US. The current activities consist of six complementary components that are discussed in the following sections.

- Organization of Existing Experimental Data for Class-C Predictions and Calibrations,
- New Complementary Laboratory Element Tests,
- System Identification Analysis of a Select Set of Existing Centrifuge Experiments,
- Class-C Predictions and Numerical Simulation of Existing Centrifuge Tests,
- Preparation for a Numerical Prediction (Validation) Exercise in a follow-on research project,
- An international workshop for planning the next phase of the project.

## 3 SCHEDULE

The US project has started in early fall 2013 and is currently in a planning phase to lay the ground for future centrifuge experimental campaign to investigate liquefaction and its consequences. It is expected that the planning phase will conclude in late 2015 and the experimental phase of the project will begin in 2016.

## 4 CURRENT STATUS OF THE PLANNING PROJECT

The planning phase of the project that is carried out by the US team is currently focused on the following efforts:

1. Prepare a preliminary set of guidelines and protocols that can be used for future comparisons of the centrifuge experiments with numerical simulations. This effort will also encompass the development of the minimum information required to present a numerical and/or constitutive model and the key stress/strain paths that should be simulated to assess the performance of a model (Kutter et al., 2014).
2. A thorough review of the centrifuge experiments documented in NEEShub and identification of a small number of high quality experiments that can be used for class-C simulations by LEAP researchers and by future modelers who intend to assess or calibrate their numerical/constitutive modeling tools (Zeghal, et al., 2014).
3. A critical review of the numerical simulation techniques, currently available and commonly used for analysis of soil liquefaction and its consequences, in order to identify the key properties required for their proper calibration.

## 5 VALIDATION CAMPAIGN: CHALLENGES AND OPPORTUNITIES

The LEAP project is an ambitious attempt to validate existing numerical and constitutive modeling approaches available for liquefaction analysis through the use of high quality experimental data that are mainly based on element tests and centrifuge modeling. The key elements of this campaign are reliable experimental data and a thorough procedure to assess the capabilities of the numerical/constitutive modeling techniques. These components are further discussed in the following sections.

### 5.1 *Availability and reliability of the experimental data*

Many existing constitutive/numerical procedures for soil liquefaction have been built based on the observed response of soil. Moreover, to use these procedures one needs to calibrate them against the results of laboratory tests. For example, to obtain the essential constants/parameters of an advanced elastoplastic model the following information may be needed:

a) Shear modulus degradation and damping characteristics of the soil at very small to small strain levels.

b) Stress-strain-strength properties of the soil at small to relatively large strain levels which are normally obtained through monotonic triaxial compression tests. To be useful for critical state based models, these experiments must be done up to a stage where the critical state can be identified.

c) Cyclic stress-strain response of the soil in drained and undrained conditions via cyclic triaxial, direct simple shear, or torsional shear tests.

All the above mentioned tests must be done on representative samples of the soil that reproduce the fabric of the soil in the centrifuge experiments which will eventually be used to assess the predictive capabilities of the model in a boundary value problem. Moreover, these tests must be repeated to establish their reliability for the use in model calibration.

The next and even more challenging step is the undertaking of centrifuge experiments that can serve as a basis for all future validation exercises. The soil specimens must be prepared with extra care to reproduce the intended boundary value problem. Here in addition to the usual precautions regarding scaling laws, use of appropriate pore fluid, and proper representation of far-field boundary conditions, one must carefully consider the proper placement of sensors, sensitivity of the sensors to the wide range of responses that soil exhibits during earthquake loading, interaction of sensors with the soil in the near liquefaction stage, difficulties in controlling the base excitation, and the consequences of the unintended movements of the soil container due to the base excitation. Centrifuge experiments must be repeated and potential sources of data scatter should be established.

### 5.2 *Assessment of constitutive models*

It is expected that a number of existing and future soil constitutive models will be used to simulate the experimental database produced by LEAP projects. A majority of existing models use plasticity (including elastoplasticity, hypoplasticity, hyperplasticity, etc.) as the main framework. However they may differ significantly in their ingredients, e.g. elastic response, yield surface, flow rule, and hardening laws. Some of these models are based on critical state soil mechanics and may require substantial experimental data (laboratory based element tests or in-situ tests) for determination and calibration of model parameters. Moreover, constitutive models that are developed for liquefiable soils may be designed to capture shear-induced volume change but may lack proper ingredients to model the volume change caused under loading with constant shear stress ratio. In some occasions, a model appears to perform well in pore pressure generation stage but is unable to reproduce

the deformations caused by soil-reconsolidation. Most models are unable to properly simulate void ratio redistribution resulting from water flow induced by excess pore pressure gradients.

Implementation of the constitutive model is also an important aspect of the entire simulation tool. Time integration of the rate equations is performed by using a variety of explicit, explicit-implicit, and fully implicit techniques. The accuracy of these techniques will have significant impact on the overall simulation results. Figure 1 shows the stress path and the stress-strain response of a soil plasticity model in an undrained cyclic triaxial test when a fully implicit integration technique is used.

Figure 2 shows the performance of the same constitutive model in the simulation of the same test when an explicit technique without strict error control is used.

Instead of a smooth stress path, this time the stress path shows locally erratic patterns due to spurious unloading during plastic loading. The spurious unloading is more visible in the  $q$ -time graph shown Figure 2(b).

Hence, it is important that this aspect of the model implementation be well documented in the course of the validation exercise envisioned in the LEAP projects. To allow for a more thorough assessment of the constitutive models used in the simulation phase of LEAP projects, it is planned to document the performance of these models in a variety of stress/strain paths, so that potential versatility as well as shortcomings of these models in the simulation of more complex boundary value problems can be correlated with their performance in the simulation of element tests.

To this end, Kutter et al. (2104) propose a set of basic experiments that a soil constitutive model needs to simulate before it is used in the simulation of boundary value problems.

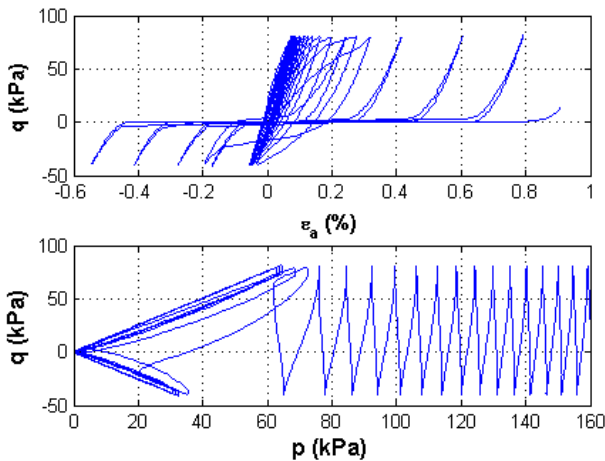
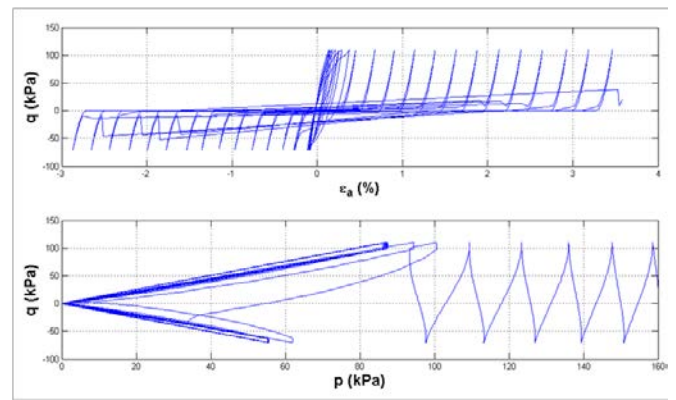
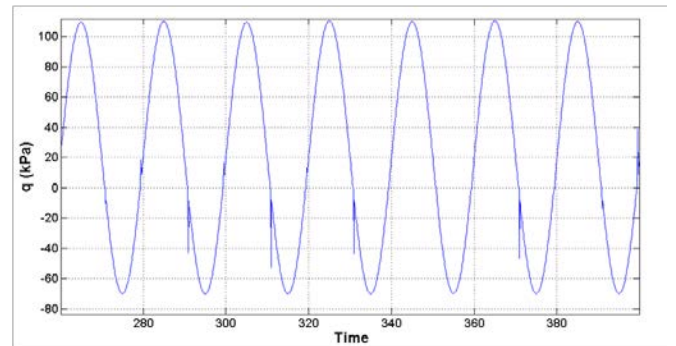


Figure 1. Performance of a soil plasticity model in simulation of an undrained cyclic triaxial shear test when a fully-implicit integration technique is used.



(a)



(b)

Figure 2. Performance of a soil plasticity model in simulation of an undrained cyclic triaxial shear test when a simplified explicit integration technique is used.

### 5.3 Assessment of numerical modeling tools

A plethora of numerical simulation platforms are now available for geotechnical engineering simulations involving liquefaction. While a majority of commonly used techniques are mesh-based (e.g., finite difference and finite element methods), there have been significant new developments on particle-based techniques such as meshfree and material point methods (MPM). In addition to these continuum-based methods, new developments in the realm of discrete element methods are also quite promising and are being used to study dynamic response of liquefiable granular soils.

Continuum-based methods are expected to be the dominant choice for the simulation phase of LEAP projects. Most available simulation tools (such as OpenSEES, FLAC, PLAXIS, etc) treat the saturated soil as two-phase media in which the differential equations governing the motion of soil and flow of pore water are formulated by using Biot's theory or mixture theory. Implementation and application of these formulations involve several key components that need to be clearly described to achieve a thorough assessment of their performance in a boundary value problem. The following issues are of particular interest to LEAP projects:

- a) The main formulation used in the numerical simulation.

- b) The element type used in finite element simulations using a particular formulation
- c) The method of time integration.

For example, Zienkiewicz and Shiomi (1984) have described three different formulations for dynamic analysis of saturated porous media, i.e.  $\mathbf{u-p}$ ,  $\mathbf{u-U}$ , and  $\mathbf{u-U-p}$ , where  $\mathbf{u}$ ,  $\mathbf{U}$ , and  $p$  represent the displacements of the soil skeleton, displacements of pore water, and pore pressure, respectively. The  $\mathbf{u-U-p}$  formulation provides the most complete formulation, but it requires a significantly larger number of unknowns per node.

Through an analysis of an elastoplastic column of soil subjected to a dynamically varying surface pressure, Zienkiewicz and Shiomi (1984) showed that for some earthquake problems the  $\mathbf{u-p}$  formulation may be able to provide solutions with sufficient accuracy. A recent work by Manzari (2014) has shown that while the results of the two methods may be close in many earthquake engineering problems, there may be significant numerical oscillations when a finite element-based  $\mathbf{u-p}$  formulation is used in liquefaction analysis. Figure 3 shows the excess pore pressure time histories computed near the ground surface in a uniform sand deposit that is subjected to 20 cycles of a sinusoidal motion with maximum amplitude of 0.25 g.

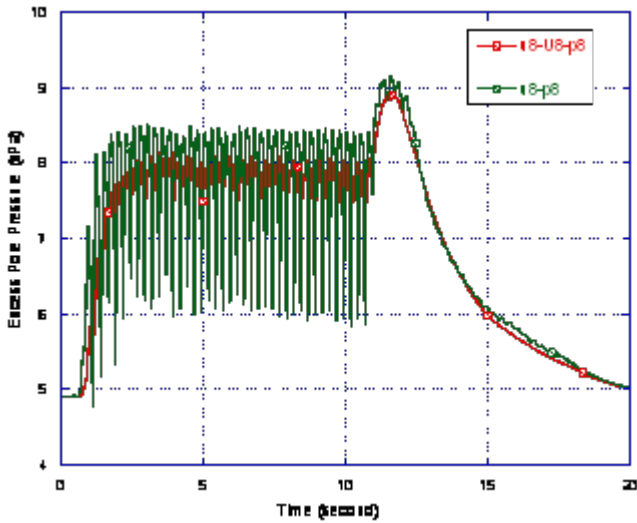


Figure 3. Comparison of two different formulations in seismic analysis of a saturated sand deposit.

Eight-noded brick elements using  $\mathbf{u-U-p}$  and  $\mathbf{u-p}$  formulations were used in the simulations. The 3D elements were constrained in the direction perpendicular to the direction of ground motion. Large oscillations are observed in the results obtained by the  $\mathbf{u-p}$  formulation. The oscillations seem to be an artifact of the numerical scheme rather than the true response of the soil.

A similar analysis for a fully submerged mildly sloping ground shows that with the  $\mathbf{u-p}$  formulation, the choice of the finite element has an impact on the simulation results. Figures 4 and 5 compare the results of seismic analyses using the  $\mathbf{u-p}$  formulation

with different finite elements. In these Figures,  $\mathbf{u4-p4}$  and  $\mathbf{u9-p4}$  are plane strain finite elements with 4 and 9 degrees of freedom for displacement of the soil skeleton and 4 degrees of freedom for pore pressure. The  $\mathbf{u8-p8}$  element is an 8-noded brick element with displacement and pore pressure degree of freedoms for all nodes. Finally the stabilized  $\mathbf{u8-p8}$  is a stabilized version of the traditional  $\mathbf{u8-p8}$  element when only one integration point is used in the analysis.

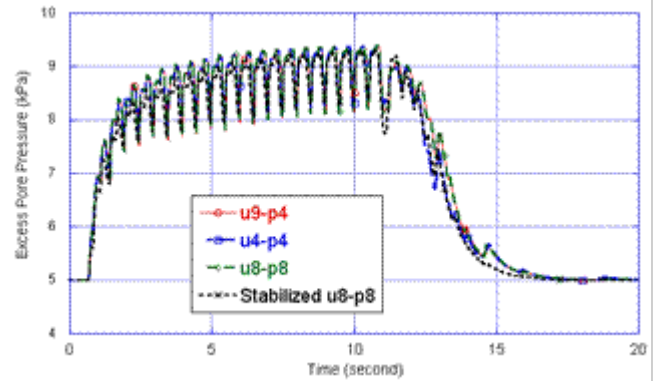


Figure 4. Comparison of pore pressure time histories near ground surface in a sloping ground; performance of different finite elements using  $\mathbf{u-p}$  formulation.

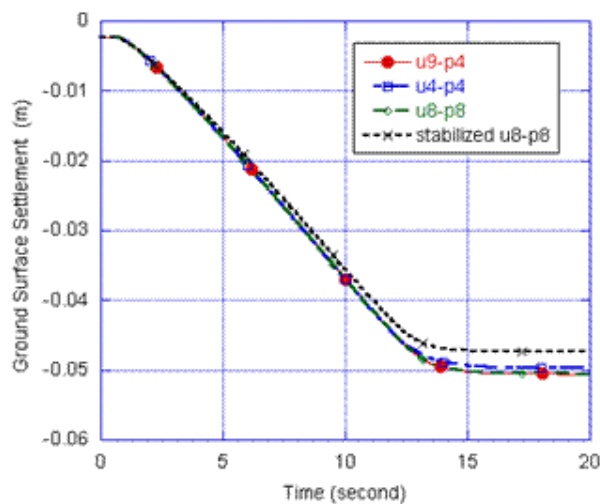
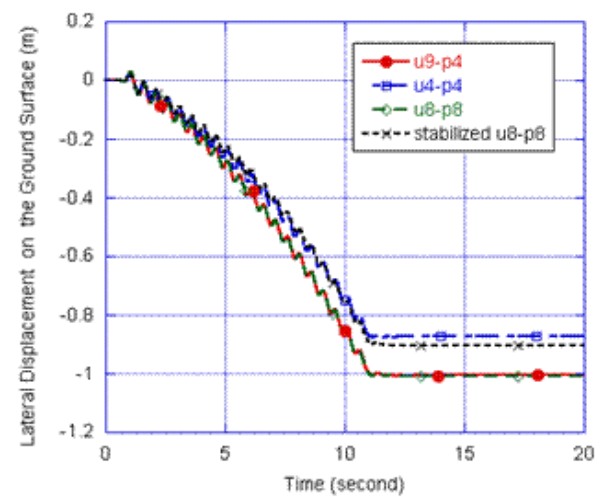


Figure 5. Comparison of the performance of different finite elements using  $\mathbf{u-p}$  formulation.

The observed differences in the computed lateral displacements are larger while the impact on computed pore water pressure is not as significant.

Given the above mentioned observations, it is important to document details of the numerical procedures used in the simulation phase of the LEAP project.

Examples of the details that need to be specified when a numerical simulation is submitted are:

- Type of the simulation platform (mesh-based, particle-based, etc.),
- General formulation used to formulate the governing equations,
- Nature of interpolation functions used in mesh-based or particle-based methods,
- Numerical scheme used in numerical integration of integrals leading to the final matrix equations,
- Numerical scheme used in time integration of the governing equations along with the parameters used,
- Return algorithms used for integration of the constitutive rate equations,
- Solution scheme used in the solution of nonlinear matrix equations.

A detailed set of information needed for submission of the numerical simulations is being developed and a preliminary set is reported by Kutter et al. (2014).

## 6 CONCLUSION

This paper presented an overview of the concept behind the *Liquefaction Analysis and Experiment Project* (LEAP), an ongoing international collaboration among researchers from the US, the UK, Japan, China, and Taiwan. The nature and goals of the project as well as the current state of the planning phase of the project in the US were discussed. Some key challenges and opportunities that face LEAP as a validation campaign for assessment of numerical/constitutive models for liquefaction analysis were presented.

## 7 ACKNOWLEDGEMENT

The planning phase of the LEAP project has been funded by the US National Science Foundation NEES research program directed by Dr. Richard Fragaszy, through the grants CMMI-1344705, CMMI-1344630, and CMMI-1344619 to the George Washington University, the University of California Davis, and Rensselaer Polytechnic Institute, respectively. This support is gratefully acknowledged.

Arulanandan, K., and R. F. Scott (1993-1994). "Verification of Numerical Procedures for the Analysis of Soil Liquefaction Problems," Proceedings of the International Conference on the Verification of Numerical Procedures for the Analysis of Soil Liquefaction Problems, Vols. 1 and 2, A. A. Balkema, Rotterdam, the Netherlands.

Bilotta, E., Lanzano, G., Madabhusi, S.P.G. and Silvestri, F., (2014), A round robin on tunnels under seismic actions, *accepted by ACTA Geotechnica*, Italy.

Byrne, P.M. (1991). "A Cyclic Shear-Volume Coupling and Pore-Pressure Model for Sand", Proceedings: Second Int. Conf. on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics, St. Louis, Paper No. 1.24, 47-55.

Byrne, P.M., Park, S.S., and Beaty, M. (2003). "Seismic liquefaction: centrifuge and numerical modeling" *Proc., 3rd International FLAC Symposium*, Sudbury, Canada.

Byrne, P.M., Park, S.S., Beaty, M.L., Sharp, M.K., Gonzalez, L., and Abdoun, T. (2004). "Numerical modeling of liquefaction and comparison with centrifuge tests". *Canadian Geotechnical Journal*, Vol. 41(2), 193-211.

Cubrinovski, M. and Ishihara, K. (1998). "State concept and modified elastoplasticity for sand modeling." *Soils and Foundations* 38(4): 213-225.

Dawson, E.M., Roth W.H., Nesarajah, S., Bureau, G., and Davis, C.A. (2001). "A Practice-Oriented Pore Pressure Generation Model," Proceedings 2nd International FLAC.

Dafalias, Y. F. and Manzari, M.T. (2004). "Simple Plasticity Sand Model Accounting for Fabric Change Effects." *ASCE Journal of Engineering Mechanics*, 130(6), 622-634.

Elgamal, A.-W., Adalier, K. Zeghal, M. (1994). "Overview and relevance of experimental data from VELACS project, models 4a,4b, and 6" *Verification of Numerical Procedures for the Analysis of Liquefaction Problems*, Arulanandan and Scott (eds.), Proceedings of Intl. Conf., Davis, CA, October 17-20, 1993, Vol. 2, A.A. Balkema, Rotterdam, The Netherlands, pp. 1681-1702.

Elgamal, A.-W., Zeghal, M., Parra, E., and Gunturi, R. (1996). "Identification and Modeling of Earthquake Ground Response, I: Site Amplification," *Soil Dynamics and Earthquake Engineering*, Vol. 15, No. 8, pp. 499-522.

Elgamal, A., Yang, Z., Parra, E., and Ragheb, A., (2003). "Modeling of Cyclic Mobility in Saturated Cohesionless Soils," *International Journal of Plasticity*, Pergamon, Elsevier Science Ltd., Vol. 19, Issue 6, pp. 883-905.

El Shamy, U., Zeghal, M., Dobry, R., Thevanayagam, S., Elgamal, A, Abdoun, T., Medina, C., Bethapudi, R., and Bennett, V. (2010). "Micromechanical Aspects of Earthquake-induced Lateral Spreading," *ASCE International Journal of Geomechanics*, v.10, p. 190.

- Finn, W.D. Liam, Yogendrakumar, M., Yoshida, N. and Yoshida, H. (1986). "TARA-3: A Program to Compute the Response of 2-D Embankments and Soil-Structure Interaction Systems to Seismic Loadings," University of British Columbia, Department of Civil Engineering, Vancouver, Canada.
- Finn, W.D. Liam and Yogendrakumar, M. (1989). "TARA-3FL: Program for Analysis of Liquefaction Induced Flow Deformations", University of British Columbia, Department of Civil Engineering, Vancouver, Canada.
- Iai, S., Matsunaga, Y., and Kameoka, T. (1992). "Strain space plasticity model for cyclic mobility," *Soils Foundations*, 32(2), pp. 1–15.
- Iai, S., Tobita, T. and Ozutsumi, O. (2011). "Induced Fabric Under Cyclic And Rotational Loads In A Strain Space Multiple Mechanism Model For Granular Materials." *International Journal for Numerical and Analytical Methods in Geomechanics*. 37: 1326–1336. doi: 10.1002/nag.2087
- Jeremic, B., Cheng, Z., Taiebat, M., and Dafalias, Y. F. (2008). "Numerical Simulation Of Fully Saturated Porous Materials." *International Journal for Numerical and Analytical Methods in Geomechanics*, 32(13), 1635–1660.
- Kutter, B. L. et al. (2014). "Proposed Outline for LEAP Verification and Validation Processes." *Proceedings of the Fourth International Conference on Geotechnical Engineering for Disaster Mitigation and Rehabilitation*, Kyoto, Japan.
- Ling, H.I. and Yang, S. (2006). "A Unified Sand Model based on Critical State and Generalized Plasticity." *Journal of Engineering Mechanics*, ASCE, 132, 1380-1391.
- Manzari, M. T. and Dafalias, Y. F. (1997). "A Critical State Two-Surface Plasticity Model for Sands." *Geotechnique*, Vol. 49, No. 2, pp. 252-272.
- Manzari, M.T. (2004). "Application of Micropolar Plasticity to Post-failure Analysis in Geomechanics Problems." *International Journal of Numerical and Analytical Methods in Geomechanics*, 28(10), 1011-1032.
- Manzari, M.T. and Regueiro, R.A. (2005). "Gradient Plasticity Modeling of Geomaterials in a Meshfree Environment, Part I: Theory and Variational Formulation." *International Journal of Mechanics Research Communications*, 32, 36-546.
- Manzari, M. T. (2009-a). "On Material versus Structural Response of Saturated Granular Soils." *DiMaggio Symposium*, in *Proceedings of the Fourth Biot Conference on Poromechanics including the second Frank L. Dimaggio Symposium*, Columbia University, June 8-10, 2009, H. I. Ling, A. Smyth, and R. Betti (eds.), DESTech Pub., pp. 1033-1039.
- Manzari, M. T. (2009-b). "An Evaluation of Liquefaction Potential of Sands at Low Confining Pressure." *Proceedings of the International Conference on Performance-Based Design in Earthquake Geotechnical Engineering, from case history to practice, (IS-Tokyo 2009)*, 15-18 June 2009, pp 1257-1260.
- Manzari, M. T. and Yonten, K. Y. (2010). "Analysis of Geostructures using a Two-Scale Constitutive Model." *Proceedings of the first International Conference on Advances in Interaction and Multiscale mechanics*, May 30-June 3, 2010, South Korea.
- Manzari, M. T. and Yonten K. (2011-a). Comparison of Two Integration Schemes for a Micropolar Plasticity Model, *Comp. Meth. Civil Eng.*, 2 (2011) 21-42.
- Manzari, M. T. and Yonten K. (2011-b), "Analysis of Post-Failure Response of Sands using a Critical State Micropolar Plasticity Model." *Interaction and Multiscale Mechanics*, Vol. 4, No. 3, 187-206.
- Manzari, M. T., Yonten, K. Y., El Ghoraiyby, M. A., and Beyzaei, C.Z. (2011). "On Analysis of Liquefaction-Induced Displacement in a Caisson Quay Wall." III ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering, M. Papadrakakis, M. Fragiadakis, V. Plevris (eds.), Corfu, Greece, 26–28 May 2011
- Manzari, M. T. and Yonten, K. Y. (2013). "C1-Finite Element Analysis of Gradient Enhanced Continua." *Journal of Mathematical and Computer Modeling*, 10.1016/j.mcm.2013.01.003.
- Manzari, M. T. (2014). "On Finite Element Analysis of Liquefaction using Elastoplasticity." *Proceedings of 17<sup>th</sup> US Congress on Theoretical and Computational Mechanics*, June 15-20, 2014.
- Perlea, V.G., and Beaty, M.H. (2010). "Corps of Engineers' Practice in the Evaluation of Seismic Deformation of Embankment Dams." *Proc., Fifth International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics*, May 24-29, San Diego. Special Lecture SPL-6, (pp. 1-30).
- Prevost, J.H. (1998). "DYNFLOW, Version 98, Release 02.A", Princeton University, Department of Civil Engineering & Op. Res., Princeton, New Jersey.
- Regueiro, R.A. and Borja, R.I. (2001). "Plane Strain Finite Element Analysis of Pressure Sensitive Plasticity with Strong Discontinuity." *Int. J. Solids Struct.*, 38(21):3647-3672.
- Taiebat, M. (2009). *Advanced Elastic-Plastic Constitutive and Numerical Modeling in Geomechanics*. LAP Lambert Academic Publishing. ISBN 978-3-8383-1123-4, 264 pages.
- Wang, Z.L. and Makdisi, F.I. (1999). "Implementing a bounding surface hypoplasticity model for sand into the FLAC program." *FLAC and Numerical Modeling in Geomechanics*, A.A. Balkema, Netherlands, 483-490.
- Zeghal at al. (2014). "LEAP: selected data for class C calibrations and class A validations." *the Fourth International Conference on Geotechnical Engineering for Disaster mitigation and Rehabilitation (4th GEDMAR)*, 16-18 September, 2014, Kyoto, Japan.
- Zienkiewicz, O. C., Chan, A. H. C., Pastor, M., Schrefler, B. A., and Shiomi, T. (1998). *Computational Geomechanics with Special Reference to Earthquake Engineering*. John Wiley and Sons, England.