Fast Multiband Spectrum Scanning for Cognitive Radio Systems

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Abstract—This paper considers the problem of how to quickly and accurately determine the availability of each spectrum band for a multi-band primary system using one or few sensors. Such problem is referred to as spectrum scanning. Two cases of practical interest are studied: 1) a single sensor case in which only one spectrum band is observed at one time; and 2) a multiple sensor case in which multiple spectrum bands are observed simultaneously. For each case, scenarios with and without a scanning delay constraint are investigated. Using mathematical tools from optimal stopping theory, optimal spectrum scanning algorithms are developed to minimize a cost function that strikes a desirable trade-off between detection performance and sensing delay. In the non delay-constrained case, it is shown that the optimal scanning algorithm is a concatenated sequential probability ratio test (C-SPRT). In the delay-constrained case, the optimal scanning algorithm has a high implementation complexity and truncation algorithms are developed as alternative low complexity options. Numerical examples are provided to illustrate the effectiveness of the proposed algorithms.

Index Terms—Cognitive radio, spectrum sensing, optimal stopping, Multiband, sequential analysis.

I. Introduction

Spectrum sensing is widely regarded as a key technology to enable dynamic spectrum access for cognitive radio (CR) systems [1]–[17]. Roughly speaking, spectrum sensing can be classified into two categories: single-band sensing and multi-band sensing. As far as spectrum sensing is concerned, the major difference between single-band sensing and multi-band sensing is that in a multi-band system, sensing needs to be performed over spectrum bands that support different primary user activities[1]. Most early work on spectrum sensing has been primarily focused on sensing a single spectrum band, including single-carrier single band [1]–[4] and multi-carrier single band [7] (e.g., single band orthogonal frequency division multiplexing (OFDM)). Recently, multi-band (multi-channel)[2] spectrum sensing has gained considerable research attention [18], [19]. More specifically, in [20] and [21], multiple narrow-band sensors (detectors), each for a spectrum band, are used to simultaneously observe multiple spectrum bands. Multi-band joint energy detection [20] and sequential detection [21] were developed to maximize overall throughput performance. When the number of candidate bands is large, these spectrum sensing schemes require a large number of sensors and joint simultaneous operation of these sensors and thus are of prohibitively large implementation complexity. In [22] and [23], a detector based on a wide-band receiver is used to collect signal samples from all multiple candidate bands. Sequential probability ratio tests (SPRT) and fixed sample size (FSS) sensing algorithms are developed to minimize multi-band sensing delay. However, a wide-band detector typically requires a high speed analog-to-digital converter (ADC) and extra signal processing elements, thus incurring additional cost/complexity.

In this paper, we consider the problem of how to determine the availability of each spectrum band in a multi-band primary system with small delay and small error probabilities using a single or few sensors. With one or a small number of sensors, secondary users (SUs) are able to observe one band or a small subset of candidate bands at a time. Two scenarios of practical interests are investigated. In the first scenario, there is a strict delay constraint on the spectrum scanning. That is, the spectrum scanning needs to be complete within a certain time period. In the second scenario, there is no strict time constraint for the scanning. That is, the spectrum scanning continues until the completion of the entire detection process. In both scenarios, our goal is to design spectrum scanning schemes that minimize a cost function that strikes a balance between error probabilities and detection delay.

We first consider a single sensor case, in which only one spectrum band is observed at a time. To minimize the scanning cost, the detector needs to design 1) *selection rules* that decide which band to collect signal samples at each time; 2) *termination rules* that decide when to terminate the entire scanning process; and 3) *terminal decision rules* that decide the availability of each band after the scanning process is complete. We first show that the problem at hand can be converted into a Markovian optimal stopping time problem [25]. Using mathematical tools from the optimal stopping theory, we derive the optimal algorithms for both

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[1]In this paper, we treat each single band as an entity in the sense that a single band no matter whether it is a single-carrier or multi-carrier single band only has two possible states: unoccupied or occupied.

[2]In this paper, we use multi-band and multi-channel interchangeably.

scenarios with and without a delay constraint. We show that in the scenario without a delay constraint, the optimal scanning algorithm is a concatenated sequential probability ratio test (C-SPRT). More specifically, we perform a SPRT test for the first band. Once the SPRT test is complete for this band, we switch to another band and carry out another SPRT on the newly switched band. The scanning process completes once all the bands have been detected by using SPRT. Hence, the scanning algorithm can be efficiently implemented. On the other hand, the implementation of the optimal algorithm for the scenario with a strict delay constraint requires large look up tables and frequent updating of posterior probabilities, thus incurring a prohibitively high computational complexity. To reduce the complexity for the delay-constrained scenario, we also propose several truncated C-SPRT algorithms that have very low implementation complexity yet are asymptotically optimal.

We next generalize the study to the multiple sensor case, in which multiple spectrum bands can be simultaneously observed. The detector again needs to design band selection rules (in this case, select a subset of bands), termination rules and terminal decision rules to minimize the cost function. The problem can also be converted into a Markovian optimal stopping time problem, and optimal rules can be derived using the tools from the optimal stopping theory. To reduce the complexity, we also design several low complexity algorithms. Extensive numerical results are presented to show the effectiveness of the proposed algorithms.

The remainder of this paper is organized as follows. Section II presents the system model and problem formulation. In Section III, optimal single observation scanning algorithms are developed for both delay-constrained and non delay-constrained scenarios. Several low complexity truncated C-SPRT schemes are also developed to reduce the implementation complexity for the delay-constrained scenario. Section IV extends the study to the multiple simultaneous observations case. In Section V, we provide extensive numerical results to illustrate the effectiveness of the proposed algorithms. Finally, in Section VI, we present several concluding remarks.

II. SYSTEM MODEL AND PROBLEM FORMULATION

In the system considered, SU can make simultaneous observations on a subset of M spectrum bands from the set $\mathcal{K}$, which consists of all K candidate spectrum bands. Let $Y_j^{(k)}$ denote the signal sample received by SU, at time j from band k. If there is no primary transmission over band k at time j, then the received signal sample $Y_j^{(k)}$ can be written as $Y_j^{(k)} = W_j^{(k)}$, in which $W_j^{(k)}$ is the background noise, whereas, if there is a primary transmission over band k at time j, then $Y_j^{(k)}$ can be written as $Y_j^{(k)} = X_j^{(k)} + W_j^{(k)}$, in which $X_j^{(k)}$ is the primary signal sample. Mathematically, the detection of the primary signals at the kth band can be formulated as a binary hypothesis testing problem as follows:

$$H_0^{(k)} : Y_j^{(k)} = W_j^{(k)}, \quad j = 1, 2, \cdots,$$
$$H_1^{(k)} : Y_j^{(k)} = X_j^{(k)} + W_j^{(k)}, \quad j = 1, 2, \cdots \quad (1)$$

where $H_0^{(k)}$ refers to the hypothesis that channel k is free and $H_1^{(k)}$ refers to the hypothesis that channel k is occupied. We

use $q_0^{(k)}(\cdot)$ to denote the density function of the signal received at the kth band when there is only noise, and use $q_1^{(k)}(\cdot)$ to denote the density function of the signal received at the kth band when there is primary signal. The algorithms developed in this paper work for any form of density functions $q_0^{(k)}(\cdot)$ and $q_1^{(k)}(\cdot)$. Furthermore, for generality, we allow the density functions to be different for different k.

Let $\pi_0^{(k)}$ denote the *a priori* probability that band k is occupied by the primary user (PU). Generally speaking, the values of $\pi_0^{(k)}$ can be different for different bands. We further assume that the status (occupied/unoccupied) of each band is independent of the status of other bands and the channel status remains unchanged during the scanning process. Our goal is to design an algorithm to decide the presence/absence of PU on each band in a way that minimizes an appropriate measure, which takes into account detection error probabilities and the sampling cost. We consider a sequential testing setup in which SU needs to detect the status of all K candidate spectrum bands. At each time j, SU tunes to a subset of bands $\mathcal{M}$ from the set $\mathcal{K}$ to make an observation, the set $\mathcal{M}$ has a size of M that represents the number of simultaneous observations. After taking one observation each from these M bands, the detector needs to decide whether to terminate the scanning or to continue. Let τ denote the *termination rule* that SU uses to decide whether or not to terminate the scanning. If SU terminates the scanning at time j, then it determines the occupancy of all bands using a *terminal decision rule* $\boldsymbol{\delta}_j = (\delta_j^{(1)}, \cdots, \delta_j^{(K)})$, in which $\delta_j^{(k)}$ takes values in $\{0, 1\}$ with 0 indicating that band k is free and 1 indicating that band k is occupied. Let $\boldsymbol{\delta} = \{\boldsymbol{\delta}_j, \ j = 1, 2, \cdots\}$ denote the sequence of decision rules used at SU. If SU chooses to continue scanning, then it uses the *band selection function* ϕ_j to select M bands from the set $\mathcal{K}$ and makes another observation from the selected band. We use $\boldsymbol{\phi} = \{\phi_j, \ j = 1, 2, \ldots, \}$ to denote the sequence of band selection functions. At the end of the scanning process, there are two types of error probabilities for band k. The first one is the false-alarm probability $P_{FA}^{(k)}$ that is the probability of declaring hypothesis $H_1^{(k)}$ is true while hypothesis $H_0^{(k)}$ is true and the second one is the misdetection probability $P_{MD}^{(k)}$ that is the probability of declaring hypothesis $H_0^{(k)}$ to be true while hypothesis $H_1^{(k)}$ is true.

Intuitively speaking, the lower $P_{FA}^{(k)}$ is, the higher number of spectrum bands are successfully identified as free and hence more secondary transmission opportunities can be exploited. Whereas, the lower $P_{MD}^{(k)}$ is, the lower the probability that SU interferes with primary transmissions. If there is no noise uncertainty [31], [32], both types of error probabilities can be made arbitrarily small by letting the number of samples go to infinity[3]. However, this will incur significant delay to reach a decision. Therefore, an appropriate cost function needs to strike a desirable tradeoff between the decision delay and the detection error probabilities. In this paper, we aim to determine the *termination rule* τ, the *terminal decision rules* $\boldsymbol{\delta}$ and the

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[3]In this paper, we do not consider noise uncertainty effects on the detection performance [31], [32]. The noise uncertainty effects in a multichannel setup will be investigated in our future work.

band selection rules ϕ that minimize the cost

$$\inf_{\tau,\boldsymbol{\delta},\boldsymbol{\phi}}\left[c\mathbb{E}\{\tau\}+\sum_{k=1}^{K}\left(c_0^{(k)}(1-\pi_0^{(k)})P_{FA}^{(k)}+c_1^{(k)}\pi_0^{(k)}P_{MD}^{(k)}\right)\right],\tag{2}$$

where $\mathbb{E}$ is expectation under the probability measure $\boldsymbol{q}_{\pi} = [q^{(1)}, q^{(2)}, \ldots, q^{(K)}]$ with $q^{(k)} := (1 - \pi_0^{(k)})q_0^{(k)} + \pi_0^{(k)}q_1^{(k)}$. The parameter c denotes the cost of unit delay, and hence the term $c\mathbb{E}\{\tau\}$ in the cost function represents the average cost of scanning delay. Similarly, $c_0^{(k)}$ denotes the cost of a false alarm event happening over band k, and $c_1^{(k)}$ denotes the cost of a misdetection event happening over band k. For generality, we allow $c_0^{(k)}$ and $c_1^{(k)}$ to be different for different bands. Clearly, the term $(1-\pi_0^{(k)})c_0^{(k)}P_{FA}^{(k)}+\pi_0^{(k)}c_1^{(k)}P_{MD}^{(k)}$ is the average cost of detection errors over band k. Hence, the cost function specified in (2) takes into consideration detection error probabilities and sampling cost, which are two key parameters closely related to the throughput of SU systems.

We note that other than the Bayesian formulation adopted in (2), one could also use a variational formulation to strike a balance between the error probabilities, namely $P_{FA}^{(k)}$ and $P_{MD}^{(k)}$ and the average delay $\mathbb{E}\{\tau\}$. More specifically, in the variational formulation, one aims to solve the following optimization problem:

$$\inf_{\tau,\boldsymbol{\delta},\boldsymbol{\phi}} \mathbb{E}\{\tau\},\tag{3}$$

s.t. $\quad P_{FA}^{(k)} \le \bar{P}_{FA}^{(k)}, \quad P_{MD}^{(k)} \le \bar{P}_{MD}^{(k)}, \quad \text{for } k = 1, \cdots, K.$

That is, we want to minimize the average delay under the constraint that the error probabilities are less than target false alarm and misdetection probabilities $\bar{P}_{FA}^{(k)}$ and $\bar{P}_{MD}^{(k)}$. However, following the same line of argument in Section 4.3 of [24], one can obtain the solution to (3) once the solution of the Bayesian formulation is found. Hence, in this paper, we focus on the Bayesian formulation (2).

Two different scenarios will be considered. In the first scenario, SU needs to stop the scanning by time T. That is, the stopping time τ is restricted to a finite interval $[0, T]$, called the delay-constrained scenario. This models the situation in which there is a strict delay constraint. In the second scenario, there is no delay constraint on the scanning time, called the non delay-constrained scenario. Relying on results from optimal stopping theory, we obtain optimal solutions for both scenarios. For convenience, a list of the mathematical symbols used is shown in Table I.

III. THE SINGLE OBSERVATION CASE

In this section, we develop scanning algorithms that solve (2) for both delay-constrained and non delay-constrained scenarios when $M = 1$, i.e., the single observation case. Hence, at time j, the detector will use the band selection rule ϕ_j to select which band to perform sensing. The results for this special case provide insights for the solution of the general case.

Let $\pi_j^{(k)}$ denote the posterior probability that band k is occupied after collecting observations up to time j. We define $\boldsymbol{\pi}_j := (\pi_j^{(1)}, \cdots, \pi_j^{(K)})$. If $\phi_j = k$, then SU selects band k to sense at time j. Via Bayesian rule, we can update the

TABLE I
LIST OF SYMBOLS

k	k^{th} band.
K	Number of bands in the mutliband setup.
$Y_j^{(k)}$	Received signal sample of band k at time j.
$W_j^{(k)}$	Additive noise.
$H_0^{(k)}$	Hypothesis test that band k is free.
$H_1^{(k)}$	Hypothesis test that band k is occupied.
$q_0^{(k)}(.)$	Noise only density function at band k.
$q_1^{(k)}(.)$	Primary signal and noise density function at band k.
$\pi_0^{(k)}$	Priori probability that band k is occupied.
$\mathcal{K}$	Set of bands $\mathcal{K} = \{1, \ldots, K\}$
$\mathcal{M}$	Set of simultaneous observation bands.
M	Number of simultaneous observations.
τ	Termination rule.
$\delta_j^{(k)}$	Terminal decision rule.
ϕ_j	Band selection rule.
$P_{FA}^{(k)}$	False alarm probability of band k.
$P_{MD}^{(k)}$	Misdetection probability of band k.
c	Cost of unit delay.
$c_0^{(k)}$	Cost of false alarm error of band k.
$c_1^{(k)}$	Cost of misdetection error of band k.
$\bar{P}_{FA}^{(k)}$	Target false alarm probability of band k.
$\bar{P}_{MD}^{(k)}$	Target misdetection probability of band k.
T	Total delay time.
$\pi_j^{(k)}$	Posterior probability that band k is occupied.
$\mathcal{F}_j$	Set of observation till time j.
$\tilde{J}_{j,T}(\mathcal{F}_j)$	Minimal expected cost-to-go function at time j.
$A_{j,T}^{(k)}$	Expected value of the cost-to-go function.
τ_{opt}	Optimal termination rule.
$U^{(k)}$	Upper bound of posterior probability in SPRT.
$L^{(k)}$	Lower bound of posterior probability in SPRT.
$\psi_{\tau}^{(k)}$	Stopping rule of the sequential test
$B_U^{(k)}$	Upper bound of the likelihood ratio.
$B_L^{(k)}$	Lower bound of the likelihood ratio.
$[\sigma^{(k)}]^2$	White Gaussian noise variance.
$P^{(k)}$	Power of the signal.
$SNR^{(k)}$	Signal to noise ratio.
$E(\tau)$	The overall average sample number (ASN) of SPRT.
FSS	Total fixed sample size of energy detector.
Q^{-1}	Inverse of the Q function.
$\Delta^{(k)}$	Saved time of uniformly added truncation at band k.

posterior probability of band k being occupied after collecting an observation $Y_j^{(k)}$ using the following equation

$$\pi_j^{(k)} = \frac{\pi_{j-1}^{(k)}q_1^{(k)}(Y_j^{(k)})}{\pi_{j-1}^{(k)}q_1^{(k)}(Y_j^{(k)}) + (1 - \pi_{j-1}^{(k)})q_0^{(k)}(Y_j^{(k)})},\tag{4}$$

where the sequence $\pi_j^{(k)}$ is evaluated recursively as in [25]. For band k that is not selected at time j, the posterior probability $\pi_j^{(k)}$ is not updated, i.e., $\pi_j^{(k)} = \pi_{j-1}^{(k)}$.

At this point, it is not clear whether or not $\boldsymbol{\pi}_j$ is a sufficient statistic for the optimization problem (2). If $\boldsymbol{\pi}_j$ is a sufficient statistic, then at time j, we can make our termination rule τ, terminal decision rule $\boldsymbol{\delta}$ and band selection rule $\boldsymbol{\phi}$ solely based on $\boldsymbol{\pi}_j$. This will greatly simplify our problem. We will show below that $\boldsymbol{\pi}_j$ is indeed a sufficient statistic for the problem under study.

We first study the optimal terminal decision rule $\boldsymbol{\delta}$. Following a standard argument in [26], it is easy to show that for any given termination rule τ and band selection rules ϕ, the

following simple terminal decision rule is optimal:

$$\delta_\tau^{(k)} = \begin{cases} 1, & \text{if } c_1^{(k)}\pi_\tau^{(k)} \geq c_0^{(k)}(1 - \pi_\tau^{(k)}), \\ 0, & \text{if } c_1^{(k)}\pi_\tau^{(k)} < c_0^{(k)}(1 - \pi_\tau^{(k)}), \end{cases} \quad (5)$$

for any $k \in \{1, \ldots, K\}$. The interpretation of this decision rule is clear. More specifically, $c_1^{(k)}\pi_\tau^{(k)}$ is the average cost of making a misdetection error. That is, we declare band k to be free while band k is busy. Similarly $c_0^{(k)}(1 - \pi_\tau^{(k)})$ is the average cost of making a false alarm error, that is we declare band k to be busy while band k is free. Thus, we declare that band k is occupied if the cost of a misdetection event is larger than that of a false alarm event, and vice visa. This result suggests that the terminal decisions can be made only based on π_j. With these terminal decision rules, the objective function in (2) is then converted into

$$\inf_{\tau, \phi} \mathbb{E}\left[c\tau + \sum_{k=1}^{K} \min\left\{ c_0^{(k)}(1 - \pi_\tau^{(k)}), c_1^{(k)}\pi_\tau^{(k)} \right\} \right]. \quad (6)$$

We will use results from optimal stopping theory [25] to solve this problem.

A. The Delay-Constrained Scenario

We first consider the scenario in which we have strict delay constraint T, i.e., we need to finish the scanning by time T. At each time instant j, SU needs to decide whether or not to terminate the scanning based on the observations that have been collected so far. Let $\mathcal{F}_j$ denote the set of observations till time j, and let $\tilde{J}_{j,T}(\mathcal{F}_j)$ denote the minimal expected cost-to-go function at time j. This is the minimal value of the expected additional cost that will incur by any strategy between time j and T. Note that $\tilde{J}_{j,T}(\mathcal{F}_j)$ is a function of $\mathcal{F}_j$, j and T. At this stage, it is not clear what the optimal strategy between time j and T is, and it is also not clear what the form of the function $\tilde{J}_{j,T}(\mathcal{F}_j)$ is. In the following, we will obtain the form of this function recursively using dynamic programming, and then obtain the optimal solution based on this function.

At first, it is clear that $\tilde{J}_{T,T} = \sum_{k=1}^{K} \min\{c_0^{(k)}(1 - \pi_T^{(k)}), c_1^{(k)}\pi_T^{(k)}\}$, since we have to stop at time T. Given $\tilde{J}_{j+1,T}(\mathcal{F}_{j+1})$, we have the following equation

$$\tilde{J}_{j,T}(\mathcal{F}_j) = \min\left\{ \sum_{k=1}^{K} \min\left\{ c_0^{(k)}(1 - \pi_j^{(k)}), c_1^{(k)}\pi_j^{(k)} \right\}, \right.$$
$$\left. c + \inf_{\phi_j} \mathbb{E}\left\{ \tilde{J}_{j+1,T}(\mathcal{F}_{j+1})|\mathcal{F}_j, \phi_j \right\} \right\}. \quad (7)$$

In this equation, the term $\sum_{k=1}^{K} \min\left\{ c_0^{(k)}(1 - \pi_j^{(k)}), c_1^{(k)}\pi_j^{(k)} \right\}$ is the additional cost that will incur if SU decides to stop scanning at time j. The term $c + \inf_{\phi_j} \mathbb{E}\left\{ \tilde{J}_{j+1,T}(\mathcal{F}_{j+1})|\mathcal{F}_j, \phi_j \right\}$ is the minimal expected additional cost that will incur if SU does not stop at time j. Note that the term $\mathbb{E}\left\{ \tilde{J}_{j+1,T}(\mathcal{F}_{j+1})|\mathcal{F}_j, \phi_j \right\}$ depends on $\mathcal{F}_j$, the observation up to time j and ϕ_j, the band selection rule. Hence, $\tilde{J}_{j,T}(\mathcal{F}_j)$ also depends on the entire observation up to time j, namely $\mathcal{F}_j$. The following lemma shows that one can greatly simplify the form of these functions.

Lemma 1: For each j, the minimal expected cost-to-go function $\tilde{J}_{j,T}(\mathcal{F}_j)$ can be written as a function of π_j, say $J_{j,T}(\pi_j)$ and the optimal band selection function ϕ_j depends only on π_j.

Proof: We will prove the lemma by induction. Clearly,

$$\tilde{J}_{T,T}(\mathcal{F}_T) = \sum_{k=1}^{K} \min\left\{ c_0^{(k)}(1 - \pi_T^{(k)}), c_1^{(k)}\pi_T^{(k)} \right\} \quad (8)$$

is a function of π_T only. Let $J_{T,T}(\pi_T)$ denote this function.

Suppose that $\tilde{J}_{j+1,T}(\mathcal{F}_{j+1})$ depends on π_{j+1} only. Let us use $J_{j+1,T}(\pi_{j+1})$ to denote it. We now show that $\tilde{J}_{j,T}(\mathcal{F}_j)$ depends on π_j only. First, we have (9), since ϕ_j admits only K possible values, the term $c + \inf_{\phi_j} \mathbb{E}\{J_{j+1,T}(\pi_{j+1})|\mathcal{F}_j, \phi_j\}$ can be written as $c + \min_{\phi_j} \mathbb{E}\{J_{j+1,T}(\pi_{j+1})|\mathcal{F}_j, \phi_j\}$. If $\phi_j = k$, then we have (10), since if we select band k, only the

$$\tilde{J}_{j,T}(\mathcal{F}_j) = \min\left\{ \sum_{k=1}^{K} \min\left\{ c_0^{(k)}(1 - \pi_j^{(k)}), c_1^{(k)}\pi_j^{(k)} \right\}, c + \inf_{\phi_j} \mathbb{E}\left\{ \tilde{J}_{j+1,T}(\mathcal{F}_{j+1})|\mathcal{F}_j, \phi_j \right\} \right\}$$
$$= \min\left\{ \sum_{k=1}^{K} \min\left\{ c_0^{(k)}(1 - \pi_j^{(k)}), c_1^{(k)}\pi_j^{(k)} \right\}, c + \inf_{\phi_j} \mathbb{E}\left\{ J_{j+1,T}(\pi_{j+1})|\mathcal{F}_j, \phi_j \right\} \right\}. \quad (9)$$

$$\mathbb{E}\{J_{j+1,T}(\pi_{j+1})|\mathcal{F}_j, \phi_j = k\} = \int J_{j+1,T}\left(\pi_j^{(1)}, \cdots, \frac{\pi_j^{(k)}q_1^{(k)}(y_{j+1}^{(k)})}{\pi_j^{(k)}q_1^{(k)}(y_{j+1}^{(k)}) + (1 - \pi_j^{(k)})q_0^{(k)}(y_{j+1}^{(k)})}, \cdots, \pi_j^{(k)} \right)$$
$$\left[\pi_j^{(k)}q_1^{(k)}(y_{j+1}^{(k)}) + (1 - \pi_j^{(k)})q_0^{(k)}(y_{j+1}^{(k)}) \right] dy_{j+1}^{(k)} := A_{j,T}^{(k)}(\pi_j), \quad (10)$$

$$\tilde{J}_{j,T}(\mathcal{F}_j) = \min\left\{ \sum_{k=1}^{K} \min\left\{ c_0^{(k)}(1 - \pi_j^{(k)}), c_1^{(k)}\pi_j^{(k)} \right\}, c + \inf_{\phi_j} \mathbb{E}\left\{ \tilde{J}_{j+1}^{T}(\mathcal{F}_{j+1})|\mathcal{F}_j, \phi_j \right\} \right\}$$
$$= \min\left\{ \sum_{k=1}^{K} \min\left\{ c_0^{(k)}(1 - \pi_j^{(k)}), c_1^{(k)}\pi_j^{(k)} \right\}, c + \min_{k} \left\{ A_{j,T}^{(k)}(\pi_j) \right\} \right\}, \quad (11)$$

posterior probability of band k will be updated. Clearly, this is a function of $\boldsymbol{\pi}_j$, and we will use $A_{j,T}^{(k)}(\boldsymbol{\pi}_j)$ to denote this function. As a result, we have (11), which is a function of $\boldsymbol{\pi}_j$ only, and we will use $J_{j,T}(\boldsymbol{\pi}_j)$ to denote this function.

Now, the optimal band selection function is given by

$$\phi_j = \arg\min \left\{ A_{j,T}^{(k)}(\boldsymbol{\pi}_j) \right\}, \tag{12}$$

which depends on $\boldsymbol{\pi}_j$. ∎

From this result, we know that $\boldsymbol{\pi}_j$ is a sufficient statistic for this problem. Without loss of optimality, we can make our decisions solely based on $\boldsymbol{\pi}_j$. Furthermore, since ϕ_j depends on $\boldsymbol{\pi}_j$ only, we have that $\{\boldsymbol{\pi}_j : j = 0, 1, \cdots\}$ forms a Markov process.

Regarding the functions $J_{j,T}(\boldsymbol{\pi}_j)$ and $A_{j,T}^{(k)}(\boldsymbol{\pi}_j)$, we have the following result. Let $\mathbf{0}$ denote a vector whose entries are all zeros and $\mathbf{1}$ denote a vector whose entries are all ones.

Lemma 2: The functions $J_{j,T}(\boldsymbol{\pi}_j)$ and $A_{j,T}^{(k)}(\boldsymbol{\pi}_j)$ are non-negative concave functions of $\boldsymbol{\pi}_j$. And $J_{j,T}(\mathbf{0}) = J_{j,T}(\mathbf{1}) = A_{j,T}^{(k)}(\mathbf{0}) = A_{j,T}^{(k)}(\mathbf{1}) = 0$.

Proof: The fact $J_{j,T}(\mathbf{0}) = J_{j,T}(\mathbf{1}) = A_{j,T}^{(k)}(\mathbf{0}) = A_{j,T}^{(k)}(\mathbf{1}) = 0$ can be shown by using a simple inductive argument.

The concavity of these functions can be shown in the same manner as [30]. ∎

These supporting lemmas show that the finite-horizon version of the optimization problem (6) can be converted to a Markov optimal stopping time problem [25]. Using the results from optimal stopping theory, we know that the optimal termination rule τ has the following form

$$\tau_{\text{opt}} = \inf \left\{ j : \sum_{k=1}^{K} \min \left\{ c_0^{(k)}(1 - \pi_j^{(k)}), c_1^{(k)} \pi_j^{(k)} \right\} \right.$$
$$\left. = c + \min_k \left\{ A_{j,T}^{(k)}(\boldsymbol{\pi}_j) \right\} \right\}. \tag{13}$$

That is, the optimal time to terminate the scanning is the time when the cost that will incur if SU decides to stop scanning, is equal to the minimal expected cost that will incur if SU does not stop.

In summary, the optimal scanning algorithm with a deadline T is described as follows:

1) Initialization: Given the maximum sensing time T, density functions $q_0^{(k)}$ and $q_1^{(k)}$, the cost of errors $c_0^{(k)}$ and $c_1^{(k)}$, we use (8), (10) and (11) to recursively compute the functions $J_{j,T}(\boldsymbol{\pi})$ and $A_{j,T}^{(k)}(\boldsymbol{\pi})$.
2) After collecting a sample, we use (4) to update the posterior probability that a selected band is being occupied.
3) Use (12) to select which band to sense if we decide to continuing sensing.
4) Use (13) to decide whether we should terminate scanning or not. If we decide to continue scanning, go back to 2). If we decide to terminate scanning, then we use decision rule (5) to decide the availability of each band.

Remark 1: In the delay-constrained scenario, the optimal algorithm involves recursive computation of $J_{j,T}(\boldsymbol{\pi})$ and $A_{j,T}^{(k)}(\boldsymbol{\pi})$, and frequent updating of the posterior probability $\pi_j^{(k)}$. These steps incur high computational complexity and

thus are the major hurdles in the implementation. We will develop several low-complexity algorithms in Section III-C based on insights gained from the non delay-constrained scenario discussed in Section III-B.

B. The Non Delay-Constrained Scenario

In this section, we consider the non delay-constrained scenario. We can obtain the optimal solution for this problem via two approaches. The first one is to let the delay constraint T defined in Section III-A go to ∞. For each T, we obtain the optimal solution as outlined in Section III-A. As T increases, the solution will converge to the optimal solution for the case with no deadline constraint. The convergence is guaranteed by Theorem 3.7 of [25]. This approach will be explained in detail in Section IV-B.

Another approach is to exploit the **decoupled** structure of the optimization problem (6). In the following, we will adopt this approach. For any stopping time τ, let $\tau^{(k)}$ be the amount of time we spend on detecting band k. We can express (6) as

$$c\mathbb{E}\left\{ \sum_{k=1}^{K} \tau^{(k)} \right\} + \sum_{k=1}^{K} \min \left\{ c_0^{(k)}(1 - \pi_\tau^{(k)}), c_1^{(k)} \pi_\tau^{(k)} \right\}$$
$$= \sum_{k=1}^{K} \left\{ c\mathbb{E}\{\tau^{(k)}\} + \min \left\{ c_0^{(k)}(1 - \pi_\tau^{(k)}), c_1^{(k)} \pi_\tau^{(k)} \right\} \right\}.$$

As a result, the quantity to be minimized is related to only the total amount of detection time. Particularly, the quantity is irrespective of sensing ordering ϕ (band selection rules). Once $\mathbb{E}\{\tau^{(k)}\} + \min \left\{ c_0^{(k)}(1 - \pi_\tau^{(k)}), c_1^{(k)} \pi_\tau^{(k)} \right\}$ is minimized for each band, the summation is also minimized. One key observation is that these K optimization problems are **independent** of each other. We can minimize each term independently. Note that this is not the case for the scenario considered in Section III-A. In Section III-A, we need to stop before time T and hence we have an additional constraint $\sum_{k=1}^{K} \tau^{(k)} \le T$, which couples these K optimization problems.

For each k, the solution that minimizes $\mathbb{E}\{\tau^{(k)}\} + \min \left\{ c_0^{(k)}(1 - \pi_\tau^{(k)}), c_1^{(k)} \pi_\tau^{(k)} \right\}$ is the well-known SPRT algorithm [25]. More specifically, for any c, $c_0^{(k)}$, $c_1^{(k)}$, $q_0^{(k)}$ and $q_1^{(k)}$, the solution is parameterized by two parameters $U^{(k)}$ and $L^{(k)}$. The stopping rule of the sequential test at the kth band is

$$\psi_\tau^{(k)} = \begin{cases} 0, & \text{if } L^{(k)} < \pi_j^{(k)} < U^{(k)}, \\ 1, & \text{otherwise}, \end{cases} \tag{14}$$

and the terminal decision rule in (5) becomes

$$\delta_\tau^{(k)} = \begin{cases} 1, & \text{if } \pi_j^{(k)} \ge U^{(k)}, \\ 0, & \text{if } \pi_j^{(k)} \le L^{(k)}. \end{cases} \tag{15}$$

The stopping rule in (14) requires the sequential test to continue if the posterior probability lies within the boundaries $U^{(k)}$ and $L^{(k)}$, and to stop sensing otherwise. Once the sequential test is stopped, a decision should be made based on the terminal decision rule of (15). That is, claiming hypothesis $H_0^{(k)}$ if $\pi_j^{(k)} \le L^{(k)}$ or claiming hypothesis $H_1^{(k)}$ if $\pi_j^{(k)} \ge U^{(k)}$. In the original Wald's SPRT [28], there is no upper limit on the number of observations required to reach a

decision. This may lead to an increase in number of samples in case of ambiguous observations. However, this problem will be treated in Section III-C by truncation as we propose different truncation schemes as an alternative to the optimal multiband delay-constrained scenario.

The optimization problem does not depend on ϕ and no delay constraint is imposed on the scanning process. Hence, with no loss of optimality, we can start scanning from band 1, once we finish scanning band 1, we switch to band 2. The entire scanning process is terminated, once we finish scanning band K. In summary, we have the following solution.

1) Initialization: Given density functions $q_0^{(k)}$ and $q_1^{(k)}$, and the costs of errors $c_0^{(k)}$ and $c_1^{(k)}$, compute parameters $L^{(k)}$ and $U^{(k)}$.
2) Starting from band 1, after taking each sample from band k, use (4) to update the posterior probability. If $\pi^{(k)} \in (L^{(k)}, U^{(k)})$, stay on band k to collect more samples. If $\pi_k \geq U^{(k)}$, claim that band k is busy, and switch to band $k+1$ to sense. If $\pi^{(k)} \leq L^{(k)}$, claim that band k is free, and switch to band $k+1$ to sense.
3) The scanning is finished, once we finish scanning band K.

Remark 2: It is clear that the optimal algorithm is a concatenation of SPRTs (C-SPRT), which is much simpler as compared with the solution for the delay-constrained scenario.

This algorithm can be further simplified for specific density functions. For example, assuming Gaussian random variables, we have $q_0(Y_j^{(k)}) = 1/\pi[\sigma^{(k)}]^2 \exp(-|Y_j^{(k)}|^2/[\sigma^{(k)}]^2)$ and $q_1(Y_j^{(k)}) = 1/\pi(P^{(k)} + [\sigma^{(k)}]^2) \exp(-|Y_j^{(k)}|^2/(P^{(k)} + [\sigma^{(k)}]^2))$. Here, $[\sigma^{(k)}]^2$ is the variance of the Gaussian noise over band k and $P^{(k)}$ is the power of the signal over band k. Let $\mathcal{S}^{(k)}$ denote the set of time slots in which we select band k to sense up to time j. Using Bayes' rule and the fact that the observations are assumed to be independent and identically distributed (i.i.d.) [26], the posterior probability can be written as

$$\pi_j^{(k)} = \frac{\pi_0^{(k)} \prod_{i \in \mathcal{S}^{(k)}} q_1^{(k)}(Y_i^{(k)})}{\pi_0^{(k)} \prod_{i \in \mathcal{S}^{(k)}} q_1^{(k)}(Y_i^{(k)}) + (1 - \pi_0^{(k)}) \prod_{i \in \mathcal{S}^{(k)}} q_0^{(k)}(Y_i^{(k)})},$$

hence $\pi_j^{(k)} > U^{(k)}$ and $\pi_j^{(k)} < L^{(k)}$ imply that

$$\frac{\prod_{i \in \mathcal{S}^{(k)}} q_1^{(k)}(Y_i^{(k)})}{\prod_{i \in \mathcal{S}^{(k)}} q_0^{(k)}(Y_i^{(k)})} > \frac{U^{(k)}(1 - \pi_0^{(k)})}{\pi_0^{(k)}(1 - U^{(k)})} := B_U^{(k)}, \text{ and}$$

$$\frac{\prod_{i \in \mathcal{S}^{(k)}} q_1^{(k)}(Y_i^{(k)})}{\prod_{i \in \mathcal{S}^{(k)}} q_0^{(k)}(Y_i^{(k)})} < \frac{L^{(k)}(1 - \pi_0^{(k)})}{\pi_0^{(k)}(1 - L^{(k)})} := B_L^{(k)}, \quad (16)$$

respectively. Let $SNR^{(k)} = P^{(k)}/[\sigma^{(k)}]^2$ denote the signal-to-noise ratio (SNR) at band k. Since $Q_0 \sim \mathcal{CN}(0, [\sigma^{(k)}]^2)$ and $Q_1 \sim \mathcal{CN}(0, P^{(k)} + [\sigma^{(k)}]^2)$, these two equations can be

further simplified as

$$\sum_{i \in \mathcal{S}^{(k)}} [Y_i^{(k)}]^2 > d^{(k)} \left(|\mathcal{S}^{(k)}| \log \left(1 + SNR^{(k)} \right) + \log B_U^{(k)} \right),$$
$$(17)$$

$$\sum_{i \in \mathcal{S}^{(k)}} [Y_i^{(k)}]^2 < d^{(k)} \left(|\mathcal{S}^{(k)}| \log \left(1 + SNR^{(k)} \right) + \log B_L^{(k)} \right),$$
$$(18)$$

in which $d^{(k)} = [\sigma^{(k)}]^2(SNR^{(k)} + 1)/SNR^{(k)}$ and $|\mathcal{S}^{(k)}|$ is the size of the set $\mathcal{S}^{(k)}$.

In general, it is difficult to obtain a closed form expressions for the boundary values $B_U^{(k)}$ and $B_L^{(k)}$ [28]. Since the optimal solution is the concatenated SPRT, we can use the approximation techniques for the SPRT to simplify the computation of $B_L^{(k)}$ and $B_U^{(k)}$. In practice, we will first specify the target error probabilities. That is, $\bar{P}_{FA}^{(k)}$ and $\bar{P}_{MD}^{(k)}$ are given. Then, using Wald's approximation [25], [28], we have

$$B_U^{(k)} = (1 - \bar{P}_{MD}^{(k)})/\bar{P}_{FA}^{(k)}, \quad B_L^{(k)} = \bar{P}_{MD}^{(k)}/(1 - \bar{P}_{FA}^{(k)}). \quad (19)$$

The actual false-alarm and misdetection probabilities of SPRT using upper- and lower-bounds in (19) are upper-bounded by the target $\bar{P}_{FA}^{(k)}$ and $\bar{P}_{MD}^{(k)}$, respectively, and more importantly the actual error probabilities become quite close to the target counterparts when $\bar{P}_{FA}^{(k)}$ and $\bar{P}_{MD}^{(k)}$ are small. In SPRT, the number of observations required to reach a decision is a random number. The average (expected) sample number (ASN) is thus considered as a useful benchmark for SPRT.

We next evaluate ASN of C-SPRT. Let us first consider the kth band. Let $\tau_l^{(k)}$ be the sample number required to reach a decision for the kth band under H_l for $l = 0, 1$. Let us define $Z_i^{(k)} := \log \left[q_1^{(k)}(Y_i^{(k)})/q_0^{(k)}(Y_i^{(k)}) \right]$ and $r^{(k)} = 1/(SNR^{(k)} + 1)$. We can readily compute $Z_i^{(k)} = \log r^{(k)} + |Y_i^{(k)}|^2/d^{(k)}$. By some straightforward computation, we have $\mu_0^{(k)} := E[Z_i^{(k)}|H_0] = \log r^{(k)} + 1 - r^{(k)}$ and $\mu_1^{(k)} := E[Z_i^{(k)}|H_1] = \log r^{(k)} + [r^{(k)}]^{-1} - 1$. Following from [25], we have $E\left[\tau_l^{(k)}|H_l\right] \approx$

$$\frac{1}{\mu_l^{(k)}} \frac{B_L^{(k)}[\exp(t_l B_U^{(k)}) - 1] + B_U^{(k)}[1 - \exp(t_l B_L^{(k)})]}{\exp(t_l B_U^{(k)}) - \exp(t_l B_L^{(k)})},$$

where $l = 0, 1$ and t_l is a nonzero constant satisfying $E[\exp(t_l Z_i^{(k)})|H_l] = 1$. It can be readily determined that t_0 is equal to 1 while t_1 is equal to -1. Clearly, the overall ASN can be expressed as

$$E(\tau) = \sum_{k=1}^{K} E\left[\tau_l^{(k)}|H_0\right](1 - \pi_0^{(k)}) + E\left[\tau_l^{(k)}|H_1\right]\pi_0^{(k)}. \quad (20)$$

To compare ASN of C-SPRT with the fixed sample size (FSS) of the energy detector, we compute the minimum FSS required to achieve target $\bar{P}_{FA}^{(k)}$ and $\bar{P}_{MD}^{(k)}$ in the multiband setup for the same density functions above. The test statistic in this case is the energy of the received signal compared to a threshold

[27]. By simple manipulations, FSS can be expressed as

$$FSS = \sum_{k=1}^{K} \left(\frac{1}{SNR^{(k)}} \left[Q^{-1}(\bar{P}_{FA}^{(k)}) \right. \right.$$
$$\left. \left. - (SNR^{(k)} + 1)Q^{-1}(1 - \bar{P}_{MD}^{(k)}) \right] \right)^2. \quad (21)$$

In summary, we have the following simplified scanning scheme:

1) Given target error probabilities $\bar{P}_{MD}^{(k)}$ and $\bar{P}_{FA}^{(k)}$, we use (19) to compute $B_L^{(k)}$ and $B_U^{(k)}$.
2) After taking a sample $Y_i^{(k)}$ from band k, we use (17) and (18) to decide whether we should skip to the next band or not. If (17) is satisfied, claim that band k is busy and skip to the next band. If (18) is satisfied, then we declare that band k is free and skip to the next band. If neither of these two is satisfied, stay on band k to observe more samples.

C. Truncated C-SPRT Schemes

As mentioned above, the complexity of the optimal solution for the delay-constrained scenario is very high. Inspired by the solution for the scenario with no delay constraint, we propose several truncated C-SPRT that can be used for the delay-constrained scenario. In the truncated C-SPRT, SPRT will be run on each band. However, a deadline will be imposed on each band. If SPRT does not finish before the deadline is reached, then it will be forced to finish, and a decision will be made using the information gathered at that time. In the following, we consider several truncation methods, namely uniform truncation, tail truncation, uniformly added truncation, and sequentially added truncation. All these truncated algorithms are asymptotically optimal as the delay constraint is relaxed. In these truncation schemes, we adopt a simple and natural scanning order (from channel 1 to channel K), which is not necessarily optimal. One can find the optimal scanning order following the steps in [29]. However, the complexity of the algorithm for finding the optimal scanning order is very high. Since our goal is to find a low-complexity but asymptotically optimal schemes, we opt to use this natural scanning order.

1) Uniform Truncation: In the uniform truncation, we need to finish detecting each band within a period of time T/K as illustrated in Fig. 1. That is, the maximal detection time is equal for all bands. In particular, the saved time for detecting a band will not be reallocated for detecting other bands. As can be seen from the above description, the advantage of uniform truncation is that no random decision will take place in the detection process, but the disadvantage of the method is that it does not utilize the saved time from the bands that can be quickly detected.

2) Tail Truncation: In the tail truncation, we need to finish the detection process within a period of time T. It implies that detection time is distributed unevenly among K bands. To be specific, the maximum detection time for the kth band is $T - \sum_{l=1}^{k-1} \tau^{(l)}$ as shown in Fig. 2. Intuitively, if T is sufficiently large, then C-SPRT with tail truncation will be able to scan all the bands, thus being able to achieve a probability similar to the
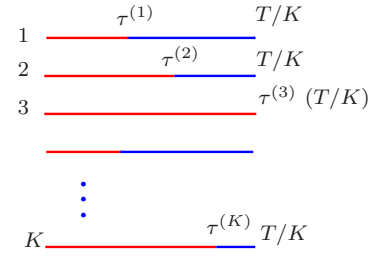

Fig. 1. An illustration of C-SPRT with the uniform truncation.
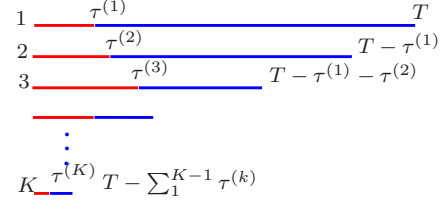

Fig. 2. An illustration of C-SPRT with the tail truncation.

one achieved by the non-truncated C-SPRT. If T is quite small, then it is highly likely that C-SPRT with tail truncation will not have time to finish detecting all K bands. In such a case, we assume that a random decision (like tossing a coin) will be made for undetected bands, thus incurring high detection errors. This is the major disadvantage of C-SPRT with tail truncation.

3) Uniformly Added Truncation: To overcome potential drawbacks of the uniform truncation and the tail truncation, we next present the uniformly added truncation. As shown in Fig. 3, we initially set the maximal detection time to be T. During the detection process, we will use the detection time saved in any detection stage to extend the maximal detection time for a later detection stage in a uniform manner. That is, the saved detection time will be added to the maximum detection time of the undetected bands equally. The maximum detection time for the kth band is $T/K + \Delta^{(k-1)}$, where $\Delta^{(k)}$ can be recursively computed as $\Delta^{(0)} = 0$ and $\Delta^{(k)} = \Delta^{(k-1)} + \left\lceil \frac{T/K + \Delta^{(k-1)} - \tau^{(k)}}{K-k} \right\rceil^+$, where $\lceil x \rceil^+$ denotes $\max\{0, x\}$. As an example, if $K = 16$, $T = 1600$ and $\tau^{(1)} = 10$, then after detecting the first band, we save 90 sample periods and will use it to equally extend the maximum detection time for the rest 15 bands. Thus, the maximum detection time for the rest bands now is 106 sample periods. By doing so, uniformly added truncation can guarantee that no random detection will take place and the detector can fully utilize available detection time.

4) Sequentially Added Truncation: In the uniformly added truncation, the saved detection time will be added to the maximum detection time of the undetected bands in a uniform manner. Apparently, when T is small, it may lead to too much truncation in an early detection stage. To amend this deficiency, we next propose an alternative truncation method, called the sequentially added truncation. The method is the same as the uniformly add truncation except that the saved time on detecting the current band will be used to extend only the maximum detection time of the next band. The maximum detection time of the kth band can be written as $T/K + \delta^{(k-1)}$, where $\delta^{(0)} = 0$ and $\delta^{(k)} = \delta^{(k-1)} + \lceil T/K - \tau^{(k)} \rceil^+$.
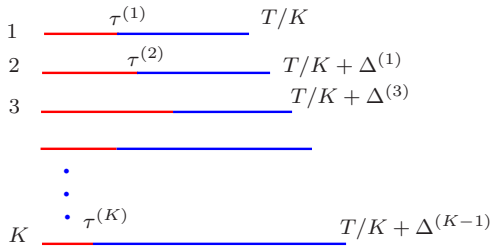
Fig. 3. An illustration of C-SPRT with the uniformly added truncation.

IV. THE MULTIPLE SIMULTANEOUS OBSERVATION CASE

In this section, we consider the case in which the sensor can take observations from more than one band at a time. Similar to Section III, we consider both delay-constrained and non delay-constrained scenarios. We discuss the two simultaneous observation case (i.e., $M = 2$) in detail. The case with more than two simultaneous observations is similar.

Again, we use $\pi_j^{(k)}$ to denote the posterior probability that band k is occupied after collecting observations up to time j. We define $\boldsymbol{\pi}_j := (\pi_j^{(1)}, \cdots, \pi_j^{(K)})$. If $\phi_j = (k_1, k_2)$, that is SU selects bands k_1 and k_2 to sense at time j, then via Bayes' rule, we can update the posterior probability of bands k_1 and k_2 being occupied after collecting observations $Y_j^{(k_1)}$ and $Y_j^{(k_2)}$ using the following equations

$$\pi_j^{(k_1)} = \frac{\pi_{j-1}^{(k_1)} q_1^{(k_1)}(Y_j^{(k_1)})}{\pi_{j-1}^{(k_1)} q_1^{(k_1)}(Y_j^{(k_1)}) + (1 - \pi_{j-1}^{(k_1)}) q_0^{(k_1)}(Y_j^{(k_1)})},$$

$$\pi_j^{(k_2)} = \frac{\pi_{j-1}^{(k_2)} q_1^{(k_2)}(Y_j^{(k_2)})}{\pi_{j-1}^{(k_2)} q_1^{(k_2)}(Y_j^{(k_2)}) + (1 - \pi_{j-1}^{(k_2)}) q_0^{(k_2)}(Y_j^{(k_2)})}.$$

For band k that is not selected at time j, the posterior probability $\pi_j^{(k)}$ is not updated, i.e., $\pi_j^{(k)} = \pi_{j-1}^{(k)}$.

We first study the optimal terminal decision rules $\boldsymbol{\delta}$. Similar to Section III, it is easy to show that the following simple terminal decision rule is optimal:

$$\delta_\tau^{(k)} = \begin{cases} 1, & \text{if } c_1^{(k)} \pi_\tau^{(k)} \geq c_0^{(k)} (1 - \pi_\tau^{(k)}), \\ 0, & \text{if } c_1^{(k)} \pi_\tau^{(k)} < c_0^{(k)} (1 - \pi_\tau^{(k)}), \end{cases} \quad (22)$$

for any $k \in \{1, \ldots, K\}$. This result suggests that the terminal decisions can be made only based on $\boldsymbol{\pi}_j$. With these terminal decision rules, the objective function in (2) under two simultaneous observations is again converted into

$$\inf_{\tau, \boldsymbol{\phi}} \mathbb{E} \left[c\tau + \sum_{k=1}^{K} \min \left\{ c_0^{(k)} (1 - \pi_\tau^{(k)}), c_1^{(k)} \pi_\tau^{(k)} \right\} \right]. \quad (23)$$

A. The Delay-Constrained Scenario

At each time instant j, SU needs to decide whether to terminate scanning based on the observations that have been collected so far. Similar to Section III, we use $\tilde{J}_{j,T}(\mathcal{F}_j)$ to denote the minimal expected cost-to-go function at time j. At first, it is clear that $\tilde{J}_{T,T} = \sum_{k=1}^{K} \min \left\{ c_0^{(k)} (1 - \pi_T^{(k)}), c_1^{(k)} \pi_T^{(k)} \right\}$, since we have to stop at time T. Given $\tilde{J}_{j+1,T}(\mathcal{F}_{j+1})$, we have (24)

and the meaning of each term is the same as that of the single-observation case.

$$\tilde{J}_{j,T}(\mathcal{F}_j) = \min \left\{ \sum_{k=1}^{K} \min \left\{ c_0^{(k)} (1 - \pi_j^{(k)}), c_1^{(k)} \pi_j^{(k)} \right\}, \right.$$
$$\left. c + \inf_{\phi_j} \mathbb{E} \left\{ \tilde{J}_{j+1,T}(\mathcal{F}_{j+1}) | \mathcal{F}_j, \phi_j \right\} \right\}. \quad (24)$$

Similar to Section III, we have the following lemma that simplifies the forms of the cost-to-go functions.

Lemma 3: For each j, the minimal expected cost-to-go function $\tilde{J}_{j,T}(\mathcal{F}_j)$ can be written as a function of $\boldsymbol{\pi}_j$, say $J_{j,T}(\boldsymbol{\pi}_j)$ and the optimal band selection function ϕ_j depends only on $\boldsymbol{\pi}_j$.

Proof: We will prove the lemma by induction. Clearly, $\tilde{J}_{T,T}(\mathcal{F}_T) = \sum_{k=1}^{K} \min \left\{ c_0^{(k)} (1 - \pi_T^{(k)}), c_1^{(k)} \pi_T^{(k)} \right\}$ is a function of $\boldsymbol{\pi}_T$ only. Let $J_{T,T}(\boldsymbol{\pi}_T)$ denote this function. Suppose that $\tilde{J}_{j+1,T}(\mathcal{F}_{j+1})$ depends on $\boldsymbol{\pi}_{j+1}$ only. Let us use $J_{j+1,T}(\boldsymbol{\pi}_{j+1})$ to denote it. We now show that $\tilde{J}_{j,T}(\mathcal{F}_j)$ depends on $\boldsymbol{\pi}_j$ only.

First, we have (25), since ϕ_j admits only $\binom{K}{2} = \frac{K(K-1)}{2}$ possible values, the term $c + \inf_{\phi_j} \mathbb{E} \{ J_{j+1,T}(\boldsymbol{\pi}_{j+1}) | \mathcal{F}_j, \phi_j \}$ can be written as $c + \min_{\phi_j} \mathbb{E} \{ J_{j+1,T}(\boldsymbol{\pi}_{j+1}) | \mathcal{F}_j, \phi_j \}$. If $\phi_j = (k_1, k_2)$, then we have (26), since if we select bands k_1 and k_2, only the posterior probability of bands k_1 and k_2 will be updated. Clearly, this is a function of $\boldsymbol{\pi}_j$ and is denoted by $A_{j,T}^{(k_1,k_2)}(\boldsymbol{\pi}_j)$. Then, we have (27) which is a function of $\boldsymbol{\pi}_j$ only, and we will use $J_{j,T}(\boldsymbol{\pi}_j)$ to denote this function. Now, the optimal band selection function is given by $\phi_j = \text{argmin} \left\{ A_{j,T}^{(k_1,k_2)}(\boldsymbol{\pi}_j) \right\}$, which depends on $\boldsymbol{\pi}_j$. ∎

From this result, we know that $\boldsymbol{\pi}_j$ is a sufficient statistic for this problem. Without loss of optimality, we can make our decisions solely based on $\boldsymbol{\pi}_j$. Furthermore, since ϕ_j depends on $\boldsymbol{\pi}_j$ only, we have that $\{\boldsymbol{\pi}_j : j = 0, 1, \cdots\}$ forms a Markov process. Regarding the functions $J_{j,T}(\boldsymbol{\pi}_j)$ and $A_{j,T}^{(k_1,k_2)}(\boldsymbol{\pi}_j)$ we have the following result similar to Lemma 2.

Lemma 4: The functions $J_{j,T}(\boldsymbol{\pi}_j)$ and $A_{j,T}^{(k_1,k_2)}(\boldsymbol{\pi}_j)$ are non-negative concave functions of $\boldsymbol{\pi}_j$. And $J_{j,T}(\mathbf{0}) = J_{j,T}(\mathbf{1}) = A_{j,T}^{(k_1,k_2)}(\mathbf{0}) = A_{j,T}^{(k_1,k_2)}(\mathbf{1}) = 0$.

Proof: The proof is similar to that for Lemma 2, and thus is omitted. ∎

These supporting lemmas show that the finite-horizon version of the optimization problem (23) can be converted to a Markov optimal stopping time problem [25]. Using the results from optimal stopping theory, we know that the optimal stopping time τ has the following form

$$\tau_{\text{opt}} = \inf \left\{ j : \sum_{k=1}^{K} \min \left\{ c_0^{(k)} (1 - \pi_j^{(k)}), c_1^{(k)} \pi_j^{(k)} \right\} \right.$$
$$\left. = c + \min_{k_1, k_2} \left\{ A_{j,T}^{(k_1,k_2)}(\boldsymbol{\pi}_j) \right\} \right\}. \quad (28)$$

That is, the optimal stopping time is the time when the cost that will incur if SU decides to stop scanning, is equal to the minimal expected cost that will incur if SU does not stop.

B. The Non Delay-Constrained Scenario

We next consider the non delay-constrained scenario. We can obtain the optimal solution by driving the constraint T in Section IV-A go to infinity as detailed in the following.

First, we have $J_{j,T+1}(\boldsymbol{\pi}) \leq J_{j,T}(\boldsymbol{\pi})$, since the set of allowed stopping time is enlarged if we allow the delay-constraint T to increase. Furthermore, we have $0 \leq J_{j,T} \leq 1$ for any j and T, and hence the following limit is well-defined: $\lim_{T\to\infty} J_{j,T}(\boldsymbol{\pi}) = \inf_{T>j} J_{j,T}(\boldsymbol{\pi}) = J_{j,\infty}(\boldsymbol{\pi})$. Also, we have $J_{j,\infty}(\boldsymbol{\pi}) = J_{j+1,\infty}(\boldsymbol{\pi})$, due to the i.i.d. nature of the observations. We will use $J(\boldsymbol{\pi})$ to denote this common function. It is easy to check that $J(\boldsymbol{\pi})$ is a concave function in $\boldsymbol{\pi}$. Furthermore, it can be shown that $J(\boldsymbol{\pi})$ is unique. By the dominated convergence theorem, the limit in (29) is well defined. Hence, $J(\boldsymbol{\pi}) = \min\{\sum_{k=1}^{K} \min\{c_0^{(k)}(1 -$

$\pi^{(k)}), c_1^{(k)}\pi^{(k)}\}, c + \min_{k_1,k_2}\{A^{(k_1,k_2)}(\boldsymbol{\pi})\}\}$. As a result, the optimal stopping rule is found in (30) and the band selection rule is $\phi_j = \operatorname{argmin}\{A^{(k_1,k_2)}(\boldsymbol{\pi}_j)\}$, which depends only on $\boldsymbol{\pi}_j$.

$$\tau_{\text{opt}} = \min\{j : \sum_{k=1}^{K} \min\left\{c_0^{(k)}(1 - \pi_j^{(k)}), c_1^{(k)}\pi_j^{(k)}\right\}$$
$$= c + \min_{k_1,k_2}\left\{A^{(k_1,k_2)}(\boldsymbol{\pi}_j)\right\}\}. \quad (30)$$

We next discuss the structure of the optimal solution and present a simple heuristic scheme. For any stopping time τ, let $\tau^{(k)}$ be the amount of time we spend on detecting band k. It is easy to see that in the optimal solution, we have $\tau = \sum_{k=1}^{K} \tau^{(k)}/2$, since we will always make observations from two bands at any time before we terminate the test. Hence, in the optimal test, when the test is terminated, there exists a set

$$\tilde{J}_{j,T}(\mathcal{F}_j) = \min\left\{\sum_{k=1}^{K} \min\left\{c_0^{(k)}(1 - \pi_j^{(k)}), c_1^{(k)}\pi_j^{(k)}\right\}, c + \inf_{\phi_j} \mathbb{E}\left\{\tilde{J}_{j+1,T}(\mathcal{F}_{j+1})|\mathcal{F}_j, \phi_j\right\}\right\}$$
$$= \min\left\{\sum_{k=1}^{K} \min\left\{c_0^{(k)}(1 - \pi_j^{(k)}), c_1^{(k)}\pi_j^{(k)}\right\}, c + \inf_{\phi_j} \mathbb{E}\left\{J_{j+1,T}(\boldsymbol{\pi}_{j+1})|\mathcal{F}_j, \phi_j\right\}\right\}. \quad (25)$$

$$\mathbb{E}\left\{J_{j+1,T}(\boldsymbol{\pi}_{j+1})|\mathcal{F}_j, \phi_j = (k_1,k_2)\right\} =$$
$$\int J_{j+1,T}\left(\frac{\pi_j^{(k_1)}q_1^{(k_1)}(y_{j+1}^{(k_1)})}{\pi_j^{(k_1)}q_1^{(k_1)}(y_{j+1}^{(k_1)}) + (1 - \pi_j^{(k_1)})q_0^{(k_1)}(y_{j+1}^{(k_1)})}, \cdots, \frac{\pi_j^{(k_2)}q_1^{(k_2)}(y_{j+1}^{(k_2)})}{\pi_j^{(k_2)}q_1^{(k_2)}(y_{j+1}^{(k_2)}) + (1 - \pi_j^{(k_2)})q_0^{(k_2)}(y_{j+1}^{(k_2)})}\right)$$
$$\left[\pi_j^{(k_1)}q_1^{(k_1)}(y_{j+1}^{(k_1)}) + (1 - \pi_j^{(k_1)})q_0^{(k_1)}(y_{j+1}^{(k_1)})\right]\left[\pi_j^{(k_2)}q_1^{(k_2)}(y_{j+1}^{(k_2)}) + (1 - \pi_j^{(k_2)})q_0^{(k_2)}(y_{j+1}^{(k_2)})\right] dy_{j+1}^{(k_1)}dy_{j+1}^{(k_2)}$$
$$:= A_{j,T}^{(k_1,k_2)}(\boldsymbol{\pi}_j), \quad (26)$$

$$\tilde{J}_{j,T}(\mathcal{F}_j) = \min\left\{\sum_{k=1}^{K} \min\left\{c_0^{(k)}(1 - \pi_j^{(k)}), c_1^{(k)}\pi_j^{(k)}\right\}, c + \inf_{\phi_j} \mathbb{E}\left\{\tilde{J}_{j+1}^{T}(\mathcal{F}_{j+1})|\mathcal{F}_j, \phi_j\right\}\right\}$$
$$= \min\left\{\sum_{k=1}^{K} \min\left\{c_0^{(k)}(1 - \pi_j^{(k)}), c_1^{(k)}\pi_j^{(k)}\right\}, c + \min_{k_1,k_2}\left\{A_{j,T}^{(k_1,k_2)}(\boldsymbol{\pi}_j)\right\}\right\}, \quad (27)$$

$$\lim_{T\to\infty} A_{j,T}^{(k_1,k_2)}(\boldsymbol{\pi}) =$$
$$\lim_{T\to\infty} \int J_{j+1,T}\left(\frac{\pi^{(k_1)}q_1^{(k_1)}(y_{j+1}^{(k_1)})}{\pi^{(k_1)}q_1^{(k_1)}(y_{j+1}^{(k_1)}) + (1 - \pi^{(k_1)})q_0^{(k_1)}(y_{j+1}^{(k_1)})}, \cdots, \frac{\pi^{(k_2)}q_1^{(k_2)}(y_{j+1}^{(k_2)})}{\pi^{(k_2)}q_1^{(k_2)}(y_{j+1}^{(k_2)}) + (1 - \pi^{(k_2)})q_0^{(k_2)}(y_{j+1}^{(k_2)})}\right)$$
$$\left[\pi^{(k_1)}q_1^{(k_1)}(y_{j+1}^{(k_1)}) + (1 - \pi^{(k_1)})q_0^{(k_1)}(y_{j+1}^{(k_1)})\right]\left[\pi^{(k_2)}q_1^{(k_2)}(y_{j+1}^{(k_2)}) + (1 - \pi^{(k_2)})q_0^{(k_2)}(y_{j+1}^{(k_2)})\right] dy_{j+1}^{(k_1)}dy_{j+1}^{(k_2)}$$
$$= \int \lim_{T\to\infty} J_{j+1,T}\left(\frac{\pi^{(k_1)}q_1^{(k_1)}(y_{j+1}^{(k_1)})}{\pi^{(k_1)}q_1^{(k_1)}(y_{j+1}^{(k_1)}) + (1 - \pi^{(k_1)})q_0^{(k_1)}(y_{j+1}^{(k_1)})}, \cdots, \frac{\pi^{(k_2)}q_1^{(k_2)}(y_{j+1}^{(k_2)})}{\pi^{(k_2)}q_1^{(k_2)}(y_{j+1}^{(k_2)}) + (1 - \pi^{(k_2)})q_0^{(k_2)}(y_{j+1}^{(k_2)})}\right)$$
$$\left[\pi^{(k_1)}q_1^{(k_1)}(y_{j+1}^{(k_1)}) + (1 - \pi^{(k_1)})q_0^{(k_1)}(y_{j+1}^{(k_1)})\right]\left[\pi^{(k_2)}q_1^{(k_2)}(y_{j+1}^{(k_2)}) + (1 - \pi^{(k_2)})q_0^{(k_2)}(y_{j+1}^{(k_2)})\right] dy_{j+1}^{(k_1)}dy_{j+1}^{(k_2)}$$
$$= \int J\left(\frac{\pi^{(k_1)}q_1^{(k_1)}(y_{j+1}^{(k_1)})}{\pi^{(k_1)}q_1^{(k_1)}(y_{j+1}^{(k_1)}) + (1 - \pi^{(k_1)})q_0^{(k_1)}(y_{j+1}^{(k_1)})}, \cdots, \frac{\pi^{(k_2)}q_1^{(k_2)}(y_{j+1}^{(k_2)})}{\pi^{(k_2)}q_1^{(k_2)}(y_{j+1}^{(k_2)}) + (1 - \pi^{(k_2)})q_0^{(k_2)}(y_{j+1}^{(k_2)})}\right)$$
$$\left[\pi^{(k_1)}q_1^{(k_1)}(y_{j+1}^{(k_1)}) + (1 - \pi^{(k_1)})q_0^{(k_1)}(y_{j+1}^{(k_1)})\right]\left[\pi^{(k_2)}q_1^{(k_2)}(y_{j+1}^{(k_2)}) + (1 - \pi^{(k_2)})q_0^{(k_2)}(y_{j+1}^{(k_2)})\right] dy_{j+1}^{(k_1)}dy_{j+1}^{(k_2)}$$
$$:= A^{(k_1,k_2)}(\boldsymbol{\pi}). \quad (29)$$

$\mathcal{K}_1 \subset \mathcal{K}$ such that $\sum_{k \in \mathcal{K}_1} \tau^{(k)} = \sum_{k \in \mathcal{K}_1^c} \tau^{(k)}$.

Using this observation, we can rewrite (23) as

$$\inf_{\tau, \phi} \left\{ c \mathbb{E} \left\{ \sum_{k=1}^{K} \frac{\tau^{(k)}}{2} \right\} + \sum_{k=1}^{K} \min \left\{ c_0^{(k)} (1 - \pi_\tau^{(k)}), c_1^{(k)} \pi_\tau^{(k)} \right\} \right\}$$

$$\text{s.t.} \quad \exists \mathcal{K}_1 \subset \mathcal{K}, \sum_{k \in \mathcal{K}_1} \tau^{(k)} = \sum_{k \in \mathcal{K}_1^c} \tau^{(k)}. \quad (31)$$

The constraint in (31) makes the problem challenging. The approach to obtain the optimal solution mentioned above (i.e., by letting $T \to \infty$) has a high computational complexity. In the following, we propose a heuristic scheme. The basic idea of the heuristic scheme is to drop the constraint in (31). This allows us to decouple the optimization problem, and enables us to derive a simple scheme that has a similar flavor as that of the single observation case discussed in Section III. More specifically, if we ignore the constraint in (31), then we have

$$c \mathbb{E} \left\{ \sum_{k=1}^{K} \tau^{(k)} / 2 \right\} + \sum_{k=1}^{K} \min \left\{ c_0^{(k)} (1 - \pi_\tau^{(k)}), c_1^{(k)} \pi_\tau^{(k)} \right\}$$

$$= \sum_{k=1}^{K} \left\{ c/2 \mathbb{E} \{\tau^{(k)}\} + \min \left\{ c_0^{(k)} (1 - \pi_\tau^{(k)}), c_1^{(k)} \pi_\tau^{(k)} \right\} \right\}.$$

As a result, the quantity to be minimized is only related to the total amount of detection time. Particularly, the quantity is irrespective of sensing ordering ϕ (band selection rules). Once $c/2 \mathbb{E} \{\tau^{(k)}\} + \min \left\{ c_0^{(k)} (1 - \pi_\tau^{(k)}), c_1^{(k)} \pi_\tau^{(k)} \right\}$ is minimized for each k, the summation is also minimized. Again, now these K optimization problems are independent of each other. Similar to Section III-B, we can minimize each term independently and obtain a heuristic solution in which SPRT is run on each band.

C. Multiple Simultaneous Observation Truncated C-SPRT Schemes

The same truncated C-SPRT schemes discussed in Section III-C could be used for the multiple observations case to reduce the complexity for the delay-constrained scenario. The uniform and tail truncation schemes are very similar to those of the single observation case. The available time T is equally divided between all bands in the uniform truncation scheme, while the total scanning time should not exceed the available time T for the tail truncation scheme. In the uniformly added truncation, the saved detection time from each sensor will be added uniformly to the maximum detection time of the undetected bands. Finally, for the sequentially added truncation scheme, the saved time from each sensor during the detection process will be added to the next available band to scan.

V. NUMERICAL EXAMPLES

In this section, we provide several numerical examples to illustrate the effectiveness of the algorithms developed in this paper. In all the examples, we assume that $\pi_0^{(k)}$ for $k = 1, \ldots, K$ is equal to $1/2$ unless indicated otherwise and the results are obtained by using Monte-Carlo simulations.

TABLE II
DETECTION PERFORMANCE OF C-SPRT FOR $K = 4, 16, 64$

SNR (dB)	0	−5	−10
P_{FA} ($K = 4$, Monte Carlo)	0.005	0.04	0.09
P_{FA} ($K = 16$, Monte Carlo)	0.005	0.04	0.09
P_{FA} ($K = 64$, Monte Carlo)	0.005	0.04	0.09
P_{MD} ($K = 4$, Monte Carlo)	0.008	0.05	0.1
P_{MD} ($K = 16$, Monte Carlo)	0.008	0.05	0.1
P_{MD} ($K = 64$, Monte Carlo)	0.008	0.05	0.1
ASN ($K = 4$, Monte Carlo)	85	305	1615
ASN ($K = 16$, Monte Carlo)	342	1222	6468
ASN ($K = 64$, Monte Carlo)	1368	4888	25875
ASN ($K = 4$, Numerical)	76	281	1548
ASN ($K = 16$, Numerical)	304	1126	6194
ASN ($K = 64$, Numerical)	1216	4502	24775
ASN ($K = 16, P_{FA} = P_{MD} = 0.1$)	161	844	6468
FSS ($K = 16, P_{FA} = P_{MD} = 0.1$)	240	1424	11600
ASN ($K = 16, P_{FA} = P_{MD} = 0.01$)	342	1990	16138
FSS ($K = 16, P_{FA} = P_{MD} = 0.01$)	784	4656	38192

Furthermore, we assume that both noise and signal are complex Gaussian.

Test Example 1: Table II compares C-SPRT in terms of false-alarm and misdetection probabilities and ASN for different SNR for $K = 4, 16, 64$ for the single observation case. In this table, we use P_{FA} and P_{MD} to denote average false-alarm and misdetection probabilities. That is, $P_{FA} = \sum_{k=1}^{K} P_{FA}^{(k)} / K$ and $P_{MD} = \sum_{k=1}^{K} P_{MD}^{(k)} / K$. As can be observed from the table, ASN increases linearly as K increases. ASN obtained by Monte-Carlo simulation is quite close to one obtained by using an approximation in (20). Table II also compares C-SPRT and energy detection in terms of the number of samples required to achieve the same false-alarm and misdetection probabilities. As can be seen from the table, C-SPRT requires much less ASN than energy detection in the cases of $(P_{FA}, P_{MD}) = (0.1, 0.1)$ and $(P_{FA}, P_{MD}) = (0.01, 0.01)$. Furthermore, it can be observed from the table that as compared to energy detection, the lower the target error probabilities are, the higher ASN savings can be achieved for C-SPRT.

Test Example 2: In this example, we compare four truncation methods in terms of the false-alarm probability, the misdetection probability for different cases of SNR, the number of bands and truncation time for the single observation case. The results of the error probabilities for three different parameter configuration cases are shown in Table III. Fig. 4 shows ASN versus SNR for the first case. Comparing the results of P_{FA} and P_{MD} in this case (i.e., $K = 16$, $T = 3560$) and ASN in Fig. 4 yields to a conclusion that when the delay constraint is strict, all truncations suffers from large detection errors when SNR is small while the non truncated C-SPRT suffers from large ASN to scan all the available bands when SNR is small. The tail truncation approach suffers a large number of random decision errors and requires the largest ASN among these truncation methods to achieve similar detection performance.

The overall performance shows that among the four truncation methods, the sequentially added truncation method yields the most desirable tradeoff between the detection performance and the detection delay. This is because the sequentially added truncation method avoids random detection and fully utilizes available detection time. In the second case, both the number of bands, K, and the allowed time, T, are increased by a
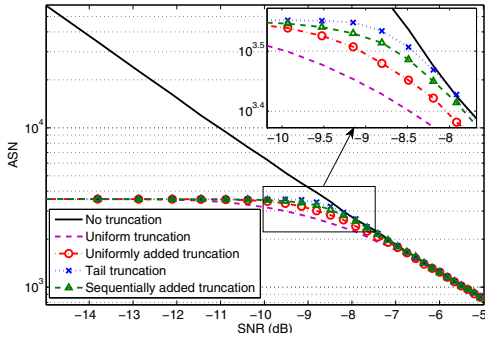
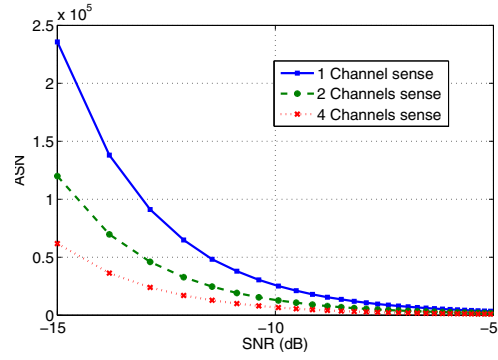Fig. 4. ASN versus SNR of the truncation C-SPRT with $K = 16$, $T = 3560$.

Fig. 5. ASN versus SNR for multiple simultaneous observations without a delay constraint.

TABLE III
DETECTION PERFORMANCE OF C-SPRT FOR DIFFERENT VALUES OF SNR

	Methods	SNR=-15 dB		SNR=-10 dB		SNR=-5 dB	
		P_{FA}	P_{MD}	P_{FA}	P_{MD}	P_{FA}	P_{MD}
	No Truncation	0.1014	0.1051	0.0917	0.0933	0.0773	0.0875
Case 1:	Uniform Truncation	0.4052	0.4262	0.2357	0.2528	0.0793	0.0913
$K = 16$,	Uniformly Added Truncation	0.4032	0.4147	0.2145	0.2341	0.0791	0.0922
$T = 3560$	Tail Truncation	0.4720	0.4670	0.2767	0.2747	0.0799	0.0933
	Sequentially Added Truncation	0.3958	0.4183	0.2139	0.2239	0.0730	0.0951
	No Truncation	0.0960	0.0983	0.0902	0.0973	0.0747	0.0933
Case 2:	Uniform Truncation	0.3986	0.4181	0.2339	0.2446	0.0745	0.0959
$K = 64$,	Uniformly Added Truncation	0.3988	0.4168	0.2201	0.2292	0.0773	0.0919
$T = 14240$	Tail Truncation	0.4693	0.4765	0.2750	0.2797	0.07916	0.0921
	Sequentially Added Truncation	0.3969	0.4159	0.2184	0.2278	0.0768	0.0936
	No Truncation	0.0962	0.0982	0.0910	0.0972	0.0751	0.0929
Case 3:	Uniform Truncation	0.2534	0.2624	0.0957	0.0997	0.0780	0.0961
$K = 64$,	Uniformly Added Truncation	0.2460	0.2535	0.0925	0.0973	0.0737	0.0931
$T = 113920$	Tail Truncation	0.3027	0.2991	0.0916	0.0980	0.0762	0.0922
	Sequentially Added Truncation	0.2427	0.2454	0.0923	0.0945	0.0739	0.0913

factor of 4. From Table III, it is clear that the performance is very similar to the first case. The results in the third case show that the average number of errors decreases considerably when the allowable delay T increases while K is fixed. The truncated schemes perform similarly to non-truncated C-SPRT when SNR increases.

Test Example 3: In this example, we illustrate the effect of multiple simultaneous observations in the non delay-constraint case. Fig. 5 shows ASN versus SNR of one band observation, two-band observations and four-band observations at one time. ASN decreases to about the half of the case of two-band observations at one time compared to one band observation and about $1/4$ for the case of four-band observations at one time.

Test Example 4: In this example, we show the effect of multiple simultaneous observations in the delay-constrained scenario. The truncation schemes are applied to the four simultaneous observations case. The results of the error probabilities are shown in Table IV. Fig. 6 shows ASN vs SNR for the four simultaneous observations case with $K = 16$ and $T = 3560$. The overall performance of the multiple observation truncation schemes shows that the sequentially added truncation and the uniformly added truncation methods have similar performance. A comparison between Fig. 6 and Fig. 4 of single observation sense shows that the critical
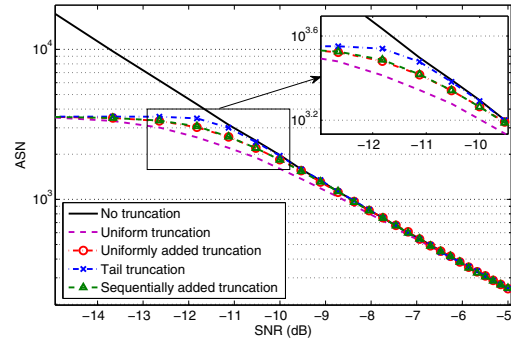
Fig. 6. ASN versus SNR for 4-simultaneous observations case with $K = 16$, $T = 3560$.

SNR under which the truncation schemes have similar ASN performance as that of the non truncation scheme is shifted by about 3 dB in the four simultaneous observation. Table V compares the error probability performance for various values of M with the same delay constraint. It is clear that by increasing the number of simultaneous observations, one can reduce the error probabilities in scanning with the same delay constraint.

TABLE IV
DETECTION PERFORMANCE OF THE 4-SIMULTANEOUS OBSERVATIONS C-SPRT

		SNR=−15 dB		SNR=−10 dB		SNR=−5 dB	
	Methods	P_{FA}	P_{MD}	P_{FA}	P_{MD}	P_{FA}	P_{MD}
Case1: $K = 16$, $T = 3560$	No Truncation	0.0921	0.0995	0.0908	0.0957	0.0750	0.0897
	Uniform Truncation	0.3131	0.3241	0.1170	0.1169	0.0762	0.0938
	Uniformly Added Truncation	0.3129	0.3247	0.1020	0.1054	0.0794	0.0958
	Tail Truncation	0.3875	0.3900	0.0943	0.0939	0.0777	0.0953
	Sequentially Added Truncation	0.3106	0.3248	0.0983	0.1039	0.0733	0.0920
Case2: $K = 64$, $T = 14240$	No Truncation	0.0958	0.1001	0.0896	0.0983	0.0758	0.0936
	Uniform Truncation	0.3157	0.3290	0.1100	0.1176	0.0765	0.0914
	Uniformly Added Truncation	0.3134	0.3243	0.0946	0.1033	0.0774	0.0954
	Tail Truncation	0.3996	0.3973	0.0904	0.0995	0.0754	0.0922
	Sequentially Added Truncation	0.3197	0.3233	0.0926	0.0996	0.0756	0.0942
Case3: $K = 64$, $T = 113920$	No Truncation	0.0942	0.0985	0.0891	0.0981	0.0773	0.0956
	Uniform Truncation	0.1196	0.1234	0.0924	0.0961	0.0775	0.0952
	Uniformly Added Truncation	0.1022	0.1056	0.0929	0.0994	0.0766	0.0921
	Tail Truncation	0.0989	0.1006	0.0917	0.0972	0.0758	0.0923
	Sequentially Added Truncation	0.0998	0.1021	0.0906	0.0982	0.0772	0.0955

TABLE V
DETECTION PERFORMANCE OF THE SINGLE AND MULTIPLE SIMULTANEOUS OBSERVATIONS C-SPRT, SNR = −10 dB

		1 band sense		2 bands sense		4 bands sense	
	Methods	P_{FA}	P_{MD}	P_{FA}	P_{MD}	P_{FA}	P_{MD}
Case1: $K = 16$, $T = 3560$	No Truncation	0.0917	0.0933	0.0865	0.1003	0.0908	0.0957
	Uniform Truncation	0.2357	0.2528	0.1613	0.1678	0.1170	0.1169
	Uniformly Added Truncation	0.2145	0.2341	0.1367	0.1432	0.1020	0.1054
	Tail Truncation	0.2767	0.2747	0.0992	0.1154	0.0943	0.0939
	Sequentially Added Truncation	0.2139	0.2239	0.1279	0.1312	0.0983	0.1039
Case2: $K = 64$, $T = 14240$	No Truncation	0.0902	0.0973	0.0906	0.0985	0.0896	0.0983
	Uniform Truncation	0.2339	0.2446	0.1594	0.1688	0.1100	0.1176
	Uniformly Added Truncation	0.2201	0.2292	0.1307	0.1364	0.0946	0.1033
	Tail Truncation	0.2750	0.2797	0.0953	0.0987	0.0904	0.0995
	Sequentially Added Truncation	0.2184	0.2278	0.1064	0.1122	0.0926	0.0996
Case3: $K = 64$, $T = 28480$	No Truncation	0.0904	0.0969	0.0906	0.0984	0.0901	0.0934
	Uniform Truncation	0.1605	0.1679	0.1072	0.1167	0.0931	0.0992
	Uniformly Added Truncation	0.1311	0.1386	0.0967	0.1052	0.0925	0.0983
	Tail Truncation	0.0942	0.1005	0.0914	0.0988	0.0903	0.0998
	Sequentially Added Truncation	0.1009	0.1097	0.0924	0.0987	0.0908	0.0990
Case4: $K = 64$, $T = 113920$	No Truncation	0.0910	0.0972	0.0898	0.0959	0.0891	0.0981
	Uniform Truncation	0.0957	0.0997	0.0927	0.0992	0.0924	0.0961
	Uniformly Added Truncation	0.0925	0.0973	0.0925	0.0991	0.0929	0.0994
	Tail Truncation	0.0916	0.0980	0.0914	0.0955	0.0917	0.0972
	Sequentially Added Truncation	0.0923	0.0945	0.0930	0.0957	0.0906	0.0982

VI. CONCLUDING REMARK

In this paper, we have investigated fast spectrum scanning algorithms for multiband CR systems using one or few narrowband detectors. Particularly, we have considered the single and multiple simultaneous observation case. For each case, both delay-constrained and non delay-constrained scenarios have been studied. Using tools from optimal stopping theory, we have developed optimal scanning algorithms that minimize cost functions that take both detection error probabilities and detection delay into consideration. For the single observation case, the optimal algorithm for the delay-constrained scenario requires large look-up tables and frequently updating of posterior probabilities, thus having a prohibitive computational complexity. In the non delay-constrained scenario with a single observation, the optimal algorithm has been shown to be C-SPRT, which can be implemented in a relatively simple

manner. We have shown that the truncated SPRT can be used as a practical alternative for the finite-horizon case. Several truncation methods have been proposed and investigated. The results have also been extended to the multiple simultaneous observations case. Extensive numerical examples have been provided to show the effectiveness of the proposed algorithms.

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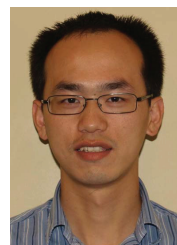
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