Optimal Sequential Channel Estimation and Probing for Multiband Cognitive Radio Systems

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***Abstract*—In this paper, we propose a novel sequential channel estimation approach for multiband cognitive radio (CR) systems. We introduce a general model and test two scenarios of practical interest. The two scenarios are as follows: 1) CR users optimally estimate all the available bands; and 2) CR users find one good channel with a large gain. In particular, we use a sequential search in which the CR users estimate the available channels one by one. During the search, the CR users determine whether to terminate the current channel estimation process and switch to the next channel based on the training symbols received so far. Our objective is to design a switch function, an estimator, and a stopping rule that minimize a combination of estimation time and error. For the multiband estimation scenario, we show that the optimal rule is to find the optimal number of symbols required for each channel in a joint optimization problem. For the good channel search problem, we show that the optimal decision rules that minimize a properly chosen cost function have a simple structure. In particular, both the termination and switching rules are threshold based. Numerical results are provided to illustrate the effectiveness of the proposed algorithms.**

***Index Terms*—Channel estimation, channel probing, cognitive radio, good channel search, multiband, optimal stopping, sequential analysis, spectrum sharing.**

I. Introduction

INCREASING the spectrum efficiency was the motivating principle for developing cognitive radio (CR) technology. Besides the high accuracy requirements for spectrum sensing, sensing delay forms a major factor in the development of spectrum sensing algorithms. This is mainly due to the fact that if less time is spent in sensing the spectrum, then more time will be available for transmission. Numerous spectrum sensing algorithms have been proposed in the literature, for example, [3]–[6], to quickly identify one or more free channels. Various approaches have been developed to not only incorporate efficient spectrum sensing techniques but also to improve the throughput of the CR system. In [7], the problem of energy allocation is studied for opportunistic spectrum access (OSA) using a multiarmed bandit framework to optimally allocate energy for sensing, probing, and data transmission. The problem of channel probing and transmission scheduling for OSA is also studied in [8]. Specifically, optimal strategies are studied to decide which channels to probe and in what order. Different approaches have been considered in the literature in order to choose a good channel that maximizes the throughput. For example, [9]–[12] demonstrate that the order in which the channels are sensed influences the throughput and that an optimal order of sensing the channels improves the throughput performance. In [5], a sensing framework is developed to optimally utilize the sensing time between the secondary users with the objective of maximizing the network throughput. Another approach is proposed in [13] to select a channel and a relay based on location information and channel usage statistics. It is important to take into account the channel quality during the spectrum sensing process to improve the throughput efficiency of multiband CR [14]–[18]. In particular, [14], [15] propose sensing and probing models to improve the CR throughput, while [17] and [18] study different scenarios to improve the quality of service (QoS) by solving an optimization problem to select a better channel. Most of the current CR research efforts focus on multiband detection under different sensing strategies to maximize the throughput [19], [20]. The optimal power allocation problem is studied in [21] and [22] with the goal of maximizing the overall CR performance.

While the field of spectrum sensing has been extensively studied, the topic of channel estimation for cognitive radios has not received much attention. Among limited existing literature on the channel estimation problem for cognitive radios, [23] proposes a sequential policy to evaluate the channel parameters in order to maximize the estimator's asymptotic efficiency. The problem of optimally placing sensing times over a time window is studied in [24]. The authors focus on obtaining the best possible estimates of the parameters of an on-off renewal channel. The focus of our paper[1] is on how to arrive at an optimal trade-off strategy between the channel estimation quality and the time left for useful data transmission. After the cognitive radios detect some channels to be free, they need to estimate the channel gain between them before they can start transmitting data. One would like the channel gain estimate to be as accurate as possible. However, this may degrade the overall throughput performance since more time is needed for the estimation process to obtain highly accurate channel gain

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[1]The results in this paper partially appear in [1], [2].

estimates. Therefore, there exists a trade-off between the quality of the channel estimation and the time left for transmission. In this paper, we study how to quickly and accurately obtain channel estimates for cognitive radios. We consider two scenarios in which we either estimate all the available channels or we find one good channel. In both cases, a tradeoff is considered between time, channel gain and estimation error. In particular, the following two cases of practical interest are studied in the context of a multiband CR setup: 1) the CR pair quickly gets accurate channel gain estimates of all free channels; and 2) the CR pair quickly and accurately searches for a channel with a large gain from the set of free channels.

In the first case, there are multiple channels that have been identified to be empty through one of the channel scanning algorithms such as the one developed in [6]. The goal is to get accurate gain estimates of these channels with a minimal number of training symbols. For this case, we consider a sequential estimation setup in which the cognitive radio pair obtains estimates of the channel gains one by one. Furthermore, the channel estimation process within a channel is also a sequential one. More specifically, instead of fixing the number of training symbols used in each channel, the transmitter keeps sending training symbols over a channel. Once the receiver obtains a reasonably good estimate of the channel gain, it sends a switch signal to the transmitter. After receiving the switch signal, the pair then proceeds to estimate the next available channel. The process terminates once the CR pair estimates the gains of all the channels. To strike a balance between the estimation error and the time required for training, we use a linear combination of the estimation variance and the total training time as the cost function. Our goal is to design switching rules and estimators that jointly minimize this cost.

In the second case, the goal is to find only one channel with a large channel gain. We again consider a sequential setup in which the CR pair sequentially inspects these free channels. When the CR pair inspects a particular channel, sequential estimation is used. After each step of the sequential estimation, the CR pair needs to make the following decisions: 1) whether to accept the channel currently under inspection as a good channel and terminate the search process; and 2) if the CR pair decides to continue the search process, the pair needs to decide whether to stay in the same channel to take more samples for more accurate channel estimation or switch to another channel in the hope of finding a better channel. Three performance quantities are of interest: the time spent on estimating the channel, the gain of the channel in which the process is terminated, and the estimation error. Clearly, the smaller is the time spent on the channel estimation process, the larger is the time left for data transmission. Similarly, the larger the channel gain and the smaller the estimation error are, the better. However, there are various tradeoffs among these quantities. Our goal is to design the optimal termination and switching rules that maximize the throughput of the CR pair. Using the theory of optimal stopping [25]–[27], we show that the optimal rules that minimize a properly chosen cost function are threshold rules. In particular, the CR pair can make the decision based on a sufficient statistic designed for this case. Once this statistic crosses a certain threshold, the CR pair will switch to another channel. Once the statistic crosses another threshold, the CR pair will terminate the search process and start data transmission over the channel.

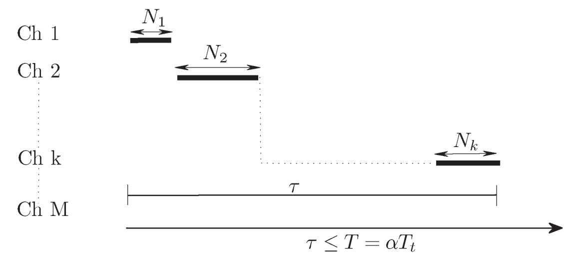

Fig. 1. System model.

The reminder of this paper is organized as follows. The system model is introduced in Section II. In Section III, we study the multiband estimation case. The sequential good channel search problem is studied in Section IV. In Section V, we provide numerical results and evaluate the performance to demonstrate the optimal solutions for all cases. Finally, concluding remarks are provided in Section VI.

II. System Model

Consider a primary communication system with multiple bands, among which M bands have been identified as unoccupied during the detection session.[2] We consider a block channel fading model between the CR transmitter and receiver, with channels being independent of each other. Furthermore, we assume uncorrelated white Gaussian noise (WGN) over each channel. During the channel estimation process, the received signal at the CR receiver for band k at each time slot j is

$$X_{(k,j)} = S_{(k,j)} h_{(k,j)} + w_{(k,j)}, \tag{1}$$

in which $S_{(k,j)}$ is the transmitted training signal from the CR transmitter in channel k with average power $\bar{S}$ and is known during the estimation session, $h_{(k,j)}$ is the channel gain coefficients with distribution $\mathcal{N} \sim (0, \sigma_k^2)$ and $w_{(k,j)}$ is i.i.d. Gaussian noise with distribution $\mathcal{N} \sim (0, \sigma_w^2)$. We denote by $\gamma = \bar{S}/\sigma_w^2$ the ratio between the power of the transmitted signal and noise. Furthermore, we assume that the channel is stationary during the estimation session.

We are interested in either estimating all the available channels or quickly searching for one good channel. In both cases, we aim to design an estimator, a switching function and a termination rule for the receiver based on the received training symbols $X_{(k,j)} = (x_{(k,1)}, x_{(k,2)} \ldots)$. Let ϕ_j denote the switching function and $\mathcal{F}_j$ the set of observations received until time j. Let $\phi_j(\mathcal{F}_j) \in \{0, 1\}$ be the switching decision function, with $\phi_j(\mathcal{F}_j) = 1$ if we decide to switch to the next channel for training and $\phi_j(\mathcal{F}_j) = 0$ if we decide to continue observing in the same channel. Furthermore, let $\bar{\phi} = \{\phi_1, \phi_2, \ldots\}$. Fig. 1 shows a typical operation of the proposed sequential estimation model, where N_k is the number of training symbols used for estimating channel k. More specifically, the CR users start

[2]In this paper, we assume that the detection is perfect. The impact of detection error will be considered in a future work.

from the first channel and continue until the whole process is terminated. Following from this, the time index j takes the following values

$$j = 1 \dots N_1, (N_1+1) \dots (N_1+N_2), \dots\dots, \left(\sum_{i=1}^{k-1} N_i + 1\right) \dots \sum_{i=1}^{k} N_i, \dots, \quad k = 1, 2, \dots\dots \tag{2}$$

Assume that the maximum training session time is upper bounded by $T = \alpha T_t$, where α is a constant $0 < \alpha < 1$ and T_t is the total transmission session time. The general Bayesian optimization problem is to determine

$$\inf_{\tau, \bar{\phi}, \hat{\mathbf{h}}} \mathbb{E}[L_\tau + c\tau], \quad \text{s.t.} \quad \tau \le T, \tag{3}$$

where $\hat{\mathbf{h}} = (\hat{h}_1, \dots, \hat{h}_k, \dots, \hat{h}_M)$ is the estimated channel gains, L_τ is the loss function related to the quality of the channel estimations, c is the cost of training symbols, and τ is the stopping time. The problem in (3) is a linear combination of the loss and cost of sampling. This formulation gives us a trade-off between loss and estimation delay under a time constraint. Since the final goal is to maximize the throughput of cognitive radios, the cost function L_τ should reflect the total throughput of CR users while being simple enough for quantitative analysis. The general form of the rate equation for channel k can be written as

$$R_{(k)} \propto \left(1 - \frac{\tau}{T_t}\right) \log_2(1 + SNR), \tag{4}$$

in which $(1 - (\tau/T_t))$ represents the fraction of the time left for data transmission.

Clearly, SNR in (4) represents the signal-to-noise ratio at the receiver side during the data transmission session. This SNR can be computed from the received signal, which depends on the transmitted signal, channel gain, and additive noise. The channel gain can be computed using any estimation technique, which will consequently result in some error from the estimation process. Assuming minimum mean-square error (MMSE) estimate and following a similar approach to [28], we can model the estimation error at channel k as

$$\check{h}_{(k,j)} = h_{(k,j)} - \hat{h}_{(k,j)}. \tag{5}$$

During the data transmission phase, the received signal will be affected by the estimation error term. Hence, we can write (1) as

$$X_{(k,j)} = S_{(k,j)} \hat{h}_{(k,j)} + \underbrace{S_{(k,j)} \check{h}_{(k,j)} + w_{(k,j)}}_{\acute{w}_{(k,j)}}, \tag{6}$$

in which $\acute{w}_{(k,j)}$ represents the sum of the received noise and the part of signal that is affected by the estimation error. Clearly, the effective SNR at the receiver side will be affected by the estimation error, which we will evaluate in Section III-A. In the meantime, we know that the rate $R_{(k)}$ is proportional to the time, gain, estimation error, and noise. Since all these factors are functions of the number of training symbols, $R_{(k)}$ is non-convex in the number of training symbols. Therefore, the use of the rate equation directly as a utility function for the sequential model will result in a hard to solve non-convex optimization problem. Furthermore, we will use a technique similar to the one developed in [28] to evaluate a lower-bound on the data rate. In the following sections, we will show how to appropriately choose the loss function that results in the desired trade-off while keeping the solution as simple as possible. We will consider two different scenarios. In the first scenario, the CR users need to estimate all the available channels quickly and accurately within a given time constraint. In the second scenario, the CR users need to quickly find one good channel with large gain without necessarily exploring all the available channels. Relying on the results from the optimal stopping theory and convex optimization, we obtain optimal solutions for both scenarios.

III. The Multiband Estimation Case

In this section, we consider the scenario in which we need to estimate the channel gains of all the available channels under a time constraint. In this case, the channel estimation problem has two effects on the overall achievable throughput. First, we want to reduce the number of symbols required by the channel training process so that more time is left for data transmission. Second, we want to reduce the channel estimation error, which, however, requires a longer training period. In this scenario, we are interested in estimating all the available channels. Hence, without loss of generality, the CR users start from band 1 and continue until the last band $k = M$, while continuing to make the switch-continue decisions based on $\phi_j(\mathcal{F}_j)$. Since we will estimate all the available channels, we are more interested in maximizing the effective SNR at the receiver side for all the channels.

Denote by $L_{(k)}$ the loss incurred in channel k during the estimation process. Our new goal is to design a switch function $\bar{\phi}$ and estimates $\hat{\mathbf{h}}$ that minimize the total cost incurred for these M channels. That is, the optimization problem in (3) can be written as

$$r = \inf_{\bar{\phi}, \hat{\mathbf{h}}} \mathbb{E}\left\{\sum_{k=1}^{M} \left[L_{(k)} + cN_k\right]\right\}, \quad \text{s.t.} \quad \sum_{k=1}^{M} N_k \le \alpha T_t, \tag{7}$$

where r is the total risk incurred after we complete the estimation process.

A. The Loss Function

To illustrate the idea, we first focus on the estimation of a single channel k. We use r_k to denote the risk incurred in channel k, i.e.,

$$r_k = L_{(k)} + cN_k. \tag{8}$$

At time j, using the channel model of (1) and choosing the average of squared error $\epsilon(\hat{h}_{(k,j)}, h_{(k,j)}) = (h_{(k,j)} - \hat{h}_{(k,j)})^2$ as the loss function (i.e., we focus on the MMSE estimate) associated with channel k

$$\begin{aligned} L_{(k,j)}\left(\hat{h}_{(k,j)}; X_{(k,j)}\right) &= \underset{h_{(k,j)}}{\mathbb{E}}\left[\epsilon\left(\hat{h}_{(k,j)}, h_{(k,j)}\right)\right] \\ &= \int_{-\infty}^{\infty} \epsilon\left(\hat{h}_{(k,j)}, h_{(k,j)}\right) \pi\left(h_{(k,j)} \mid x_{(k,j)}\right) dh_{(k,j)} \\ &= \operatorname{var}\left(h_{(k,j)} \mid x_{(k,j)}\right), \end{aligned} \tag{9}$$

where $\pi(h_{(k,j)} \mid x_{(k,j)})$ is the posterior density of $h_{(k,j)}$ after taking observations until time j and $\operatorname{var}(h_{(k,j)} \mid x_{(k,j)})$ is the conditional variance of $h_{(k,j)}$. In the sequel, we will show that the conditional variance of the MMSE estimator in this problem does not depend on the observations. From this, we conclude that the problem will reduce to a fixed sample size problem.

Since we only care about the number of observations in each channel for this case, we will drop the notation of time index j from our analysis in the reminder of this section. We can easily evaluate the resulting MMSE estimation $\hat{h}_k$, which is the conditional mean, and $\operatorname{var}(h_k \mid x_k)$, which is the mean square error [29]. For the general model, the conditional mean and covariance after N_k observations are

$$\begin{aligned} \mathbb{E}(\mathbf{h}_k \mid \mathbf{x}_k) &= \boldsymbol{\mu}_{h_k} + \left(\mathbf{C}_{h_k}^{-1} + \mathbf{S}^T\mathbf{C}_w^{-1}\mathbf{S}\right)^{-1}\mathbf{S}^T\mathbf{C}_w^{-1}\left(\mathbf{x}_k - \mathbf{S}\boldsymbol{\mu}_{h_k}\right), \\ \mathbf{C}_{h_k|x_k} &= \left(\mathbf{C}_{h_k}^{-1} + \mathbf{S}^T\mathbf{C}_w^{-1}\mathbf{S}\right)^{-1}, \end{aligned}$$

where $\boldsymbol{\mu}_{h_k}$ is the mean vector and $\mathbf{C}_{h_k}$ is the covariance matrix of h_k, i.e., $\boldsymbol{\mu}_{h_k} = 0$ and $\mathbf{C}_{h_k} = \sigma_k^2$, since we are interested in a scalar version of the channel gain h_k. Furthermore, $\mathbf{x}_k$ and $\mathbf{S}$ are the received and transmitted signals of size $[N_k \times 1]$ respectively and $\mathbf{C}_w$ is covariance matrix of noise w, i.e., $\mathbf{C}_w = \sigma_w \mathbf{I}$ where $\mathbf{I}$ is $[N_k \times N_k]$ identity matrix. For the PDFs given in Section II and assuming $\mathbf{S} = \mathbf{1}$ is known at the CR receiver, we have

$$\begin{aligned} E(h_k \mid \mathbf{x}_k) &= \left(\frac{1}{\sigma_k^2} + \mathbf{1}^T\frac{\mathbf{I}}{\sigma_w^2}\mathbf{1}\right)^{-1}\mathbf{1}^T\frac{\mathbf{I}}{\sigma_w^2}\mathbf{x}_k, \\ \mathbf{C}_{h_k|x_k} &= \left(\frac{1}{\sigma_k^2} + \mathbf{1}^T\frac{\mathbf{I}}{\sigma_w^2}\mathbf{1}\right)^{-1}. \end{aligned}$$

After some simplifications, the conditional mean and covariance, i.e., the variance $\operatorname{var}(h_k \mid x_k)$ in this case, can be evaluated as

$$\hat{h}_k = \frac{\sigma_k^2}{1 + N_k\gamma\sigma_k^2}\sqrt{\frac{\gamma}{\sigma_w^2}}\sum_{i=1}^{N_k} x_{(k,i)}, \tag{10}$$

$$\operatorname{var}(h_k \mid x_k) = \frac{\sigma_k^2}{1 + N_k\gamma\sigma_k^2}. \tag{11}$$

The estimation error $\check{h}_k = h_k - \hat{h}_k$ is a Gaussian random variable with zero mean and variance $\operatorname{var}(h_k \mid x_k)$. In (6), we have shown that the received symbols will suffer additional error that can be added to the original WGN. For simplicity, we assume the average power of the transmitted signal $\bar{S} = 1$ for all of the upcoming analysis. Hence, $X_k = h_k + \acute{w}_k$ and $\acute{w}_k = \check{h}_k + w$. After completing the estimation process, we have the mean square error

$$\mathbb{E}\left[\check{h}_k^2\right] = \mathbb{E}_{h_k}[\epsilon(\hat{h}_k, h_k)] = \operatorname{var}(h_k \mid x_k),$$

which represents the average power added by the error term. Denoted by $\sigma_{eff(k)}^2$, the variance of $\acute{w}_k$ termed the effective error-noise variance, is defined as

$$\sigma_{eff(k)}^2 = \mathbb{E}\left[\check{h}_k^2\right] + \mathbb{E}[w^2] = \frac{\sigma_k^2}{1 + N_k\gamma\sigma_k^2} + \sigma_w^2. \tag{12}$$

From (6), we can evaluate the effective SNR over channel k as

$$SNR_e^{(k)} = \frac{|\hat{h}_k|^2}{\sigma_{eff(k)}^2}. \tag{13}$$

The effective SNR, i.e., $SNR_e^{(k)}$, represents the SNR at the receiver side during the data transmission session, which we will use to compute the rate $R_{(k)}$ in (4).

The loss function $L_{(k)}$ in (9) becomes

$$L_{(k)} = \frac{\sigma_k^2}{1 + N_k\gamma\sigma_k^2},$$

and the total risk is given by

$$r_k = \frac{\sigma_k^2}{1 + N_k\gamma\sigma_k^2} + N_k c.$$

If we ignore the time constraint in (7), the optimal number of symbols N_k that minimizes the risk can simply be found by setting $(dr_k)/(dN_k) = 0$, from which we obtain

$$\lceil N_k^* \rceil = \frac{1}{\sqrt{\gamma c}} - \frac{1}{\gamma\sigma_k^2}, \tag{14}$$

where $\lceil N_k^* \rceil$ is the smallest integer that is greater than or equal to N_k^*. To simplify the notation, we will drop the ceiling symbol and always consider N_k^* to be equal to a positive integer or zero. From (14), the cost c must be less than $\gamma[\sigma_k^2]^2$ to make $N_k > 0$. However, this solution is for one channel with no time constraint and we will solve the general multiband case with time constraint in the following subsection.

B. Multiband Joint Optimization Problem

The problem of multiband is formulated as a joint optimization problem that minimizes the total risk in (7). The estimate $\hat{h}_k$, the variance $\operatorname{var}(h_k \mid x_k)$, and the effective error-noise variance $\sigma_{eff(k)}^2$ over channel k are shown in (10), (11), and (12), respectively. The objective of the joint optimization problem is to minimize the total risk jointly over all the available channels subject to a time constraint or, equivalently, to maximize the data transmission rate. Then, the optimization problem in (7) reduces to

$$\begin{aligned} \text{Minimize} \quad & r = \sum_{k=1}^{M}\left(\frac{\sigma_k^2}{1 + N_k\gamma\sigma_k^2} + cN_k\right), \\ \text{s.t.} \quad & \sum_{k=1}^{M} N_k \leq \alpha T_t. \end{aligned} \tag{15}$$

The objective function in (15) is convex in N_k. Let $\alpha_{eff} = \tau/T_t$. Notice that minimizing over N_k is equivalent to minimizing over α_{eff}, since α provides an upper bound for α_{eff}. Using convex optimization techniques to solve this problem, we first write the Lagrangian function as

$$\mathcal{L}(\lambda, N_k) = \sum_{k=1}^{M} \left(\frac{\sigma_k^2}{1 + N_k \gamma \sigma_k^2} + cN_k \right) + \lambda \left(\sum_{k=1}^{M} N_k - \alpha T_t \right).$$

The KKT conditions are

$$\frac{\partial \mathcal{L}(\lambda, N_k)}{\partial N_k} = \frac{-\left(\sigma_k^2\right)^2 \gamma}{[1 + N_k \gamma \sigma_k^2]^2} + c + \lambda = 0, \tag{16a}$$

$$\lambda \left(\sum_{k=1}^{M} N_k - \alpha T_t \right) = 0, \tag{16b}$$

$$\sum_{k=1}^{M} N_k - \alpha T_t \le 0, \tag{16c}$$

$$\lambda \ge 0, \tag{16d}$$

$$N_k \ge 0, k = 1, 2, \ldots M. \tag{16e}$$

On solving these equations, we have

$$N_k^* = \frac{1}{\gamma} \left(\sqrt{\frac{\gamma}{\lambda + c}} - \frac{1}{\sigma_k^2} \right), \tag{17a}$$

$$\lambda^* = \frac{\gamma}{\left[\frac{\alpha T_t \gamma}{M} + \frac{1}{M} \sum_{m=1}^{M} \frac{1}{\sigma_m^2} \right]^2} - c, \tag{17b}$$

$$N_k^* = \frac{\alpha T_t}{M} + \frac{1}{M\gamma} \sum_{m=1}^{M} \frac{1}{\sigma_m^2} - \frac{1}{\gamma \sigma_k^2}. \tag{17c}$$

To simplify the notation, define ρ as

$$\rho = \left[\frac{\alpha T_t \gamma}{M} + \frac{1}{M} \sum_{m=1}^{M} \frac{1}{\sigma_m^2} \right].$$

After examining the condition in (16b), we find that if $\lambda > 0$, then the second term is equal to zero and the solution in (17c) for N_k satisfies the optimality conditions for $c \le \gamma/\rho^2$. On the other hand, if $c \ge \gamma/\rho^2$, then λ should equal zero to satisfy (16b) and (16d), and the optimal solution for N_k^* is

$$N_k^* = \frac{1}{\sqrt{\gamma c}} - \frac{1}{\gamma \sigma_k^2}. \tag{18}$$

To summarize, the optimal solution for N_k in (17c) and (18) represents the optimal number of symbols for each channel that minimizes the total risk for *region one* when $c \le \gamma/\rho^2$ and for *region two* when $c \ge \gamma/\rho^2$, respectively. The solutions for both regions are equal at the critical point $c = \gamma/\rho^2$. The total minimized risk for each region is found as

$$r_{t1} = \frac{M}{\rho} + c\alpha T_t, \quad c \le \frac{\gamma}{\rho^2}, \tag{19a}$$

$$r_{t2} = 2M\sqrt{\frac{c}{\gamma}} - \frac{c}{\gamma} \sum_{k=1}^{M} \frac{1}{\sigma_k^2}, \quad c \ge \frac{\gamma}{\rho^2}. \tag{19b}$$

It is obvious that the optimal number of symbols per channel for *region one* changes with $\gamma\sigma_k^2$ proportionally, since the solution first divides the total time uniformly, and then it allocates some of the time according to channel quality, i.e., it allocates more time for a better channel. In *region two*, unlike *region one*, the solution depends on the cost c directly, since the effect of the cost is more than the effect of channel quality when the cost per symbol is relatively high. The optimal number of symbols decreases inversely with $\sqrt{c}$, but still gives more time for better channels through the term $\gamma\sigma_k^2$. On the other hand, as the constraint α is more relaxed, the critical point $c = \gamma/\rho^2$ shifts to the left.

To analyze the data transmission performance of the optimized problem, we first compute α_{eff} as

$$\alpha_{eff} = \alpha, \quad c \le \frac{\gamma}{\rho^2},$$

$$\alpha_{eff} = \frac{1}{T_t} \left(\frac{M}{\sqrt{\gamma c}} - \sum_{k=1}^{M} \frac{1}{\gamma \sigma_k^2} \right), \quad c \ge \frac{\gamma}{\rho^2},$$

which represents the total number of symbols. Therefore, the optimal stopping time in (15) becomes $\tau_{opt} = \alpha_{eff} T_t$. The switching rule is given by

$$\phi_j = \begin{cases} 1, & j = N_1^*, N_1^* + N_2^*, \ldots, \sum_{k=1}^{M} N_k^*, \\ 0, & \text{Otherwise.} \end{cases} \tag{20}$$

The optimal solution results in an equal mean square error $\mathrm{var}(h_k \mid x_k)$ for all the channels and we denote it by e. The channel gain estimate and the effective error-noise variance are given by

$$\sigma_{eff}^2 = e + \sigma_w^2,$$

$$\hat{h}_k = \left(1 - \frac{e}{\sigma_k^2} \right) \frac{\beta_k}{\sqrt{\gamma \sigma_w^2}},$$

where β_k is the sample mean of the kth channel. The sample mean β_k and the error term e can be evaluated as

$$\beta_k = \frac{1}{N_k^*} \sum_{i=1}^{N_k^*} x_{(k,i)},$$

$$e = \frac{1}{\frac{\alpha T_t \gamma}{M} + \frac{1}{M} \sum_{k=1}^{M} \frac{1}{\sigma_k^2}}, \quad c \le \frac{\gamma}{\rho^2},$$

$$e = \sqrt{\frac{c}{\gamma}}, \quad c \ge \frac{\gamma}{\rho^2}.$$

The lower-bound on the data transmission rate for M parallel channels is

$$R = (1 - \alpha_{eff}) \frac{1}{2} \sum_{k=1}^{M} \log_2 \left(1 + SNR_e^{(k)} \right).$$

IV. The Case of Sequential Good Channel Search

In this section, we investigate the second case; that is, how to find one good channel with a large gain. Based on the discussion in Section II, we aim to choose an appropriate loss function for the Bayesian formulation. Hence, we start from (4) for the rate $R_{(k)}$ of any CR pair, which depends on the effective SNR. In order to maximize the rate of a CR pair, we want to minimize the estimation time and maximize $SNR_e^{(k)}$. The signal power $\bar{S} = 1$ is assumed to be equal for all channels. Investigating $SNR_e^{(k)}$ in (13), there could be different combinations of reward/loss functions that comply with the sequential nature of the problem. However, we want to choose a loss function that would simplify the mathematical analysis. For simplicity, assume that the channel coefficient h_k is real. The analysis involving a set of complex channel coefficients is similar. Let the function $L_{(k,j)}$ be the loss encountered in band k at time j. Let τ be the stopping time that we stop sampling and choose channel k to be a good channel. Then, the optimization problem of (3) will take the following form

$$\inf_{k\leq M,\phi,\hat{\mathbf{h}}} \mathbb{E}[L_\tau + c\tau] \quad \text{s.t.} \quad \tau \leq T. \tag{21}$$

The more the number of samples that we observe, the better the estimate that we get and the smaller the loss $L_{(k,j)}$ will be. However, extra observation will cost extra (c) per symbol. Thus, we choose our loss function for this problem as follows

$$L_{(k,j)}\left(\hat{h}_{(k,j)}; X_{(k,j)}\right) = -\frac{\hat{h}^2_{(k,j)}}{\underset{h_{(k,j)}}{\mathbb{E}}\left[\epsilon\left(\hat{h}_{(k,j)}, h_{(k,j)}\right)\right] + \sigma_w^2}. \tag{22}$$

Clearly, the loss function in (22) is the instantaneous form of the effective SNR in (13) with a negative sign. From (9) we know that the term $\mathbb{E}_{h_{(k,j)}}[\epsilon(\hat{h}_{(k,j)}, h_{(k,j)})]$ does not depend on the observation $x_{(k,j)}$ for all $j \geq 1$. However, the estimate $\hat{h}^2_{(k,j)}$ depends on the observations and hence the loss function in (22) indeed depends on the observations. Thus, the problem does not yield a fixed sample size.

Since the estimation process within a channel is sequential in nature, we will use a sequential linear minimum mean square error estimator (LMMSE). We derive our sequential estimator by starting from the general sequential LMMSE model developed in [29] for the Bayesian linear model. Assuming a scalar version of (1) and after some manipulations, we can write the sequential channel estimate and the estimation error variance as

$$\hat{h}_{(k,j+1)} = \frac{\hat{h}_{(k,j)}}{1+\gamma\sigma^2_{(k,j)}} + \frac{\sigma^2_{(k,j)}}{1+\gamma\sigma^2_{(k,j)}} x_{(k,j+1)} \sqrt{\frac{\gamma}{\sigma_w^2}}, \tag{23}$$

$$\sigma^2_{(k,j+1)} = \frac{\sigma^2_{(k,j)}}{1+\gamma\sigma^2_{(k,j)}}. \tag{24}$$

Clearly, we can iteratively compute the channel gain estimate $\hat{h}_{(k,j)}$ and the mean square error $\sigma^2_{(k,j)}$ at each time instant j. For the sequential estimator, we need to calculate the initializations before we start estimating in a new channel for values of $j = 0, N_1, N_1 + N_2, \ldots, \sum_{i=1}^k N_i$, namely

$$\hat{h}_{(k,j)} = \mathbb{E}(h_k) = 0, \quad \sigma^2_{(k,j)} = \sigma_k^2. \tag{25}$$

Similar to the multiband case in Section III, the received symbols will suffer additional effect from the estimation error during the data transmission session. Hence, the instantaneous effective error-noise variance is given by

$$\sigma^2_{eff(k,j)} = \sigma^2_{(k,j)} + \sigma^2_w. \tag{26}$$

It is clear that (23) and (24) are the sequential form of (10) and (11), respectively. Using the sequential estimator of (23), we want to form an optimization problem that finds a good channel quickly and accurately. The whole process will be terminated if we claim the current channel to be a good one or until the time j reaches $T = \alpha T_t$. Therefore, our next step is to simplify the sequential loss function in (22).

A. *The Sequential Loss Function*

To simplify the analysis, we will find a recursion version of the loss function. Substituting (23) and (24) for $j+1$ into (22) and after some manipulations, we have

$$L_{(k,j+1)} = a_{(k,j)}L_{(k,j)} + b_{(k,j)}\hat{h}_{(k,j)}x_{(k,j+1)} + d_{(k,j)}x^2_{(k,j+1)}, \tag{27}$$

where

$$\mu_{(k,j)} = \frac{\frac{\sigma^2_{(k,j)}}{\sigma^2_w}}{1+\frac{\sigma_w^2}{\sigma^2_{(k,j)}}}\gamma, \quad a_{(k,j)} = \frac{\mu_{(k,j+1)}}{\mu_{(k,j)}},$$
$$b_{(k,j)} = -2\frac{\mu_{(k,j+1)}}{\sqrt{\frac{\gamma}{\sigma_w^2}}\sigma^2_{(k,j)}}, \quad d_{(k,j)} = -\mu_{(k,j+1)}.$$

The loss function in (27) represents the recursion of loss if we continue in the same channel. Clearly, if we switch to another channel, the initial loss will be reset and we will get the observations from the next channel $k+1$. That is, the switching rule $\phi_j = 1$ will switch the system to the next channel and initialize the estimate and the variance from (25) for $j = 0, N_1, N_1+N_2, \ldots, \sum_{i=1}^k N_i$, i.e. $\hat{h}_{(k+1,j)} = 0$ and $\sigma^2_{(k+1,j)} = \sigma^2_{(k+1)}$. Thus, from (27), the loss of switch will take the form $d_{(k+1,j+1)}x^2_{(k+1,j+1)}$. Finally, the loss function is

$$\begin{aligned} L_{(k,j+1)} = &\Big[a_{(k,j)}L_{(k,j)} + b_{(k,j)}\hat{h}_{(k,j)}x_{(k,j+1)} \\ &+ d_{(k,j)}x^2_{(k,j+1)}\Big]\mathbb{1}(\phi_j=0) + \Big[d_{(k+1,j+1)}x^2_{(k+1,j+1)} \\ &\Big|\Big(j = 0, N_1, N_1+N_2, \ldots, \sum_{i=1}^k N_i\Big)\Big]\mathbb{1}(\phi_j=1). \end{aligned} \tag{28}$$

The notation $\mathbb{1}(.)$ represents the Boolean indicator function, which takes a value of 1 if the associated event occurs, and 0 otherwise. To further simplify the problem, we will assume that the channels are identical with noise variance equal to σ^2, and therefore we will drop the notion for channel index k in the analysis. Hence, we write the simplified form of (28) as

$$\begin{aligned} L_{j+1} = &\left[a_j L_j + b_j \hat{h}_j x_{j+1} + d_j x^2_{j+1}\right]\mathbb{1}(\phi_j = 0) \\ &+ \left[d_1 x^2_{j+1}\right]\mathbb{1}(\phi_j = 1), \end{aligned} \tag{29}$$

where

$$d_1 = \left[d_{(k+1,j+1)} \middle| \left(j = 0, N_1, N_1 + N_2, \ldots, \sum_{i=1}^{k} N_i \right) \right].$$

It is important to emphasize that the problem does not have a preferred channel probing order since the channels are assumed to be identical. In a nutshell, we will start from band one and continue until a decision is made or until we reach the last band. If we finish the last band without a decision, then we will switch to band one and continue in the same manner until a decision is made. Hence, the loss function could be viewed as a random sequence that re-initializes if we decide to switch and take the observations from the next band.

B. Sequential Search Optimal Stopping

To solve the optimization problem in (21), consider the loss function in (29). At each time j, we want to decide whether to stop sampling or to continue based on $\mathcal{F}_j$. Let $\tilde{V}_j^T(\mathcal{F}_j)$ be the minimal expected cost-to-go function. Obviously, the cost incurred if we stop sampling at time j is L_j, while the expected cost that will incur if we continue sampling is given by

$$c + \inf_{\phi_j} \mathbb{E}\left\{ \tilde{V}_{j+1}^T(\mathcal{F}_{j+1}) | \mathcal{F}_j, \phi_j \right\}.$$

This cost represents the smaller value between the cost if we continue sampling in the same channel, and the cost if we switch to the next channel and abandon the observations up to time j.

Given $\tilde{V}_{j+1}^T(\mathcal{F}_{j+1})$, we can write the cost function as

$$\tilde{V}_j^T(\mathcal{F}_j) = \min\left\{ L_j, c + \min\left\{ \mathbb{E}\left\{ \tilde{V}_{j+1}^T(\mathcal{F}_{j+1}) | \mathcal{F}_j, \phi_j = 0 \right\}, \mathbb{E}\left\{ \tilde{V}_{j+1}^T(\mathcal{F}_{j+1}) | \mathcal{F}_j, \phi_j = 1 \right\} \right\} \right\}. \quad (30)$$

The function $\tilde{V}_j^T(\mathcal{F}_j)$ represents the smallest among the three costs at each time j. Following this, we have Lemma 1 that simplifies the solution.

Lemma 1: For each time j, the minimal expected cost-to-go function $\tilde{V}_j^T(\mathcal{F}_j)$ can be written as a function of $(L_j, \hat{h}_j)$ only and we write it as $V_j^T(L_j, \hat{h}_j)$.

Proof: First, at time T we have $\tilde{V}_T^T(\mathcal{F}_T) = L_T$ is a function of L_T only and we write as $V_T^T(L_T) = L_T$. For each time j, $0 \le j \le T-1$, suppose that $\tilde{V}_{j+1}^T(\mathcal{F}_{j+1})$ is a function of L_{j+1} only and we write it as $V_{j+1}^T(L_{j+1})$. Consequently, (30) becomes

$$\tilde{V}_j^T(\mathcal{F}_j) = \min\left\{ L_j, c + \min\left\{ \int V_{j+1}^T(L_{j+1}) f_c(x_{j+1}|\mathcal{F}_j) \times dx_{j+1}, \int V_{j+1}^T(L_{j+1}) f_s(x_{j+1}|\mathcal{F}_j) dx_{j+1} \right\} \right\}. \quad (31)$$

Clearly, $f_c(x_{(j+1)}|\mathcal{F}_j)$ is the conditional density of $x_{(j+1)}$ if we continue sampling within the same channel given what has already been observed. This density is given by $\mathcal{N}(\hat{h}_j, \sigma_j^2 + \sigma_w^2)$. Similarly, $f_s(x_{(j+1)}|\mathcal{F}_j)$ is the conditional density of $x_{(j+1)}$ if we decide to switch to the next channel and abandon the previous observations, which is given by $\mathcal{N}(0, \sigma^2 + \sigma_w^2)$. The second term in (31) is a function of $(L_j, \hat{h}_j)$ only and we denote it by $Ac_j^T(L_j, \hat{h}_j)$. That is,

$$Ac_j^T(L_j, \hat{h}_j) = \int \tilde{V}_{j+1}^T\Big(a_j L_j + b_j \hat{h}_j x_{j+1} + d_j x_{j+1}^2 \Big) f_c(X_{j+1}|\hat{h}_j) dx_{j+1}. \quad (32)$$

The third term of (31) is a constant independent of $(L_j, \hat{h}_j)$ and we denote it by As_j^T, since a switch will reset the loss and is given by

$$As_j^T = \int \tilde{V}_{j+1}^T\left(d_1 x_{j+1}^2 \right) f_s(X_{j+1}) dx_{j+1}. \quad (33)$$

Obviously, $\tilde{V}_j^T(\mathcal{F}_j)$ is indeed a function of $(L_j, \hat{h}_j)$ only, and we write it as $V_j^T(L_j, \hat{h}_j)$. Based on this discussion, we have

$$V_T^T(L_T) = L_T, \quad \text{at } j = T. \quad (34)$$

Then, recursively for $j = 1, \ldots, T-1$, we compute

$$V_j^T(L_j, \hat{h}_j) = \min\left\{ L_j, c + \min\left\{ Ac_j^T(L_j, \hat{h}_j), As_j^T \right\} \right\}. \quad (35)$$

■

In summary, we will use dynamic programming to solve this problem using (29), (34), and (35). In the following lemma, we study the functions $Ac_j^T(L_j, \hat{h}_j)$ and $V_j^T(L_j, \hat{h}_j)$.

Lemma 2: The functions $Ac_j^T(L_j, \hat{h}_j)$ and $V_j^T(L_j, \hat{h}_j)$ are negative concave in $(L_j, \hat{h}_j)$.

Proof: Refer to Appendix A. ■

Remark 1: Following from Lemma 1 and Lemma 2, we have converted the optimization problem in (21) into a Markov optimal stopping time problem, with the optimal stopping rule being of a threshold type [30].

Remark 2: The function $V_j^T(L_j, \hat{h}_j)$ is symmetric with respect to $\hat{h}$, meaning that $V_j^T(L_j, \hat{h}_j) = V_j^T(L_j, -\hat{h}_j)$ and we write it as $V_j^T(L_j)$.

Clearly, $V_j^T(L_j) \le L_j$ and Fig. 2 shows a representation of the cost function in terms of L_j. The optimal stopping time is given by $\tau_{opt} = \inf\{j : L_j < L_c^*\}$, where L_c^* is given by

$$L_c^* = c + \min\left\{ Ac_j^T(L_c^*), As_j^T \right\}. \quad (36)$$

The switching rule is given by

$$\phi_j = \begin{cases} 1, & \text{if } L_j > L_s^*, \\ 0, & \text{if } L_c^* < L_j < L_s^*, \end{cases} \quad (37)$$

where L_s^* is found by

$$L_s^* = \sup\left\{ L_j : Ac_j^T(L_j) < As_j^T \right\}. \quad (38)$$

In summary, we have the following solution. Given the initial problem constraints, we first find the two threshold values

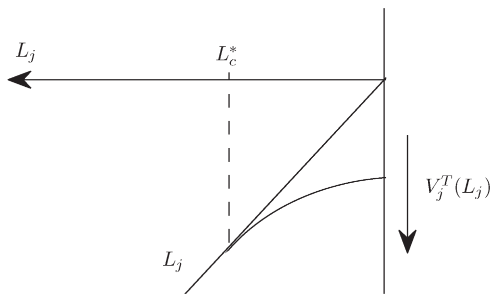

Fig. 2. Cost versus loss representation.

(L_s^*, L_c^*) offline. We use these threshold values to find the good channel. If the loss is higher than L_s^*, we switch to the next channel. If the loss becomes less than L_c^*, we stop and claim the current channel as good. Otherwise, we continue the search for a good channel.

V. Simulation Results

In this section, we provide several numerical examples to evaluate the performance and demonstrate the optimal solutions for both cases.

A. *Numerical Results for the Multiband Estimation Case*

In all of the following simulations, we assume $M = 16$, meaning that 16 channels were identified as free during the detection session. The value of γ is assumed to be fixed during the training and data transmission sessions, meaning that no power allocation is performed. Channel prior variances are assumed to be *complex normal* and are selected randomly between 0.1 and 1 to test the performance of the proposed model. In these tests, we compare the performance of our scheme with that of a uniform solution in which one divides the available time αT_t equally between all the channels. We set $T = 10\,000$ and $\alpha = 0.15$. Note that larger values of alpha will only further degrade the performance of the uniform model, since the time is fixed for each channel. Therefore, we choose a reasonable value of α that guarantees a fair comparison between the uniform allocation strategy and our proposed method for a broad range of γ values.

Fig. 3 depicts the risk of the proposed model against the uniform model for different values of the cost c and two values of γ. In the uniform model, the risk versus the cost relation is almost linear, since τ is fixed and the variance term in (15) is very small. Hence, the cost term $c\tau$ dominates. In general, the risk increases as the cost increases and is higher for lower values of γ. The rate of risk increase is much lower in the optimized case due to the fact that the proposed optimization strategy keeps the risk minimized as the cost increases. At lower values of c, the performances of the optimized and uniform models are very close since the optimized model uses all the available time αT_t in *region one*. Comparing the cases of $\gamma = 0$ dB and $\gamma = -10$ dB, the critical point $\gamma/\rho^2 = c$ is shifted up in the case of $\gamma = -10$ dB. Intuitively speaking, this is expected since at lower values of γ, the model consumes more time to perform

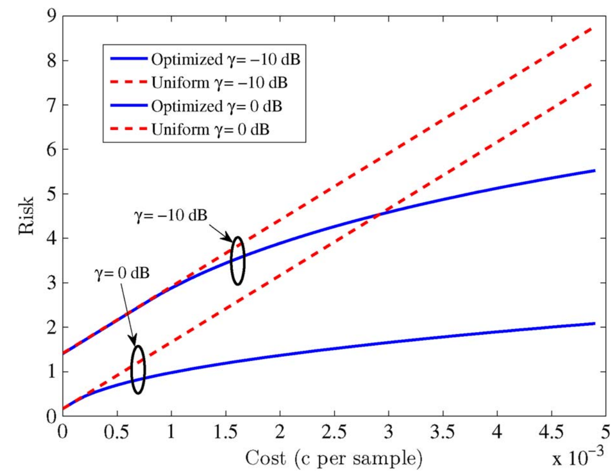

Fig. 3. Risk vs. cost for $\gamma = 0, -10$ dB.

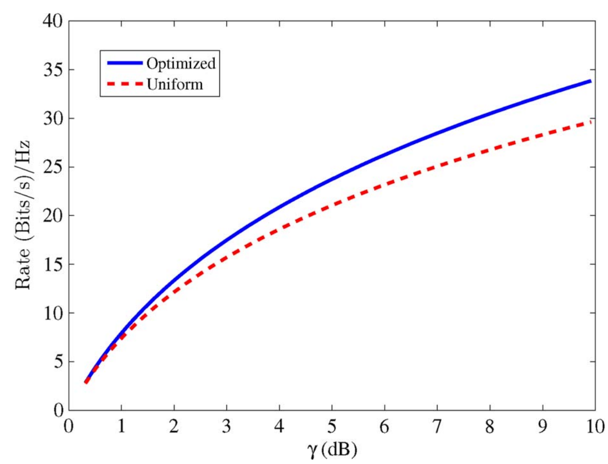

Fig. 4. Transmission rate vs. γ, $c = 0.001$.

the estimation and, consequently, utilizes all the available time until it reaches the critical point.

After demonstrating the risk optimization strategy in Fig. 3, we will now investigate the performance of our solution for the data transmission to see the effect of risk minimization on the transmission rate. A Monte-Carlo simulation is performed with 10 000 repetitions. Fig. 4 shows the transmission rate of the optimized and uniform cases versus γ, with $c = 0.001$ per sample. Clearly, the rate is maximized due to the effect of minimizing the risk and the number of symbols. To be more specific, the total value of the risk was jointly optimized between all channels to maximize the rate. Furthermore, it can also be noticed from Fig. 4 that as γ increases, the improvement in the optimized rate over that of the uniform rate also increases. For example, at $\gamma = 5$ dB, the improvement in the rate is 12.85%, while at $\gamma = 10$ dB, the improvement is 14.28%. The reason for the improvement lies in the fact that there is a substantial reduction in τ value when the symbols are less noisy. The overall transmission rate gain is due to the combined effect of optimization between the channels and the reduction in the total number of symbols τ.

In the next test, we investigate the case when the cost is decreased to $c = 0.0001$ per sample. Fig. 5 represents the total average sample number (ASN), i.e., the value of τ, for

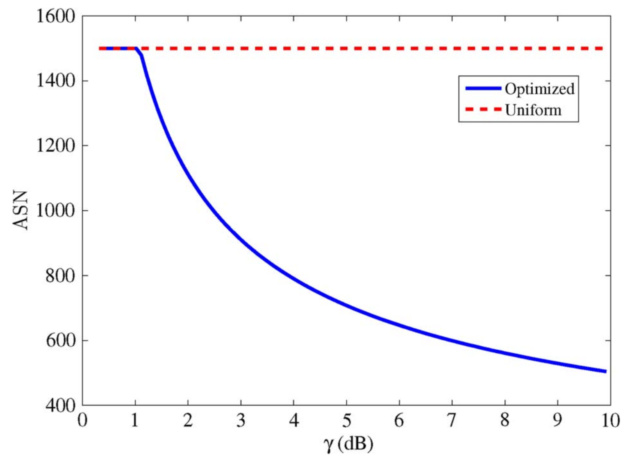

Fig. 5. ASN vs. γ, $c = 0.0001$.

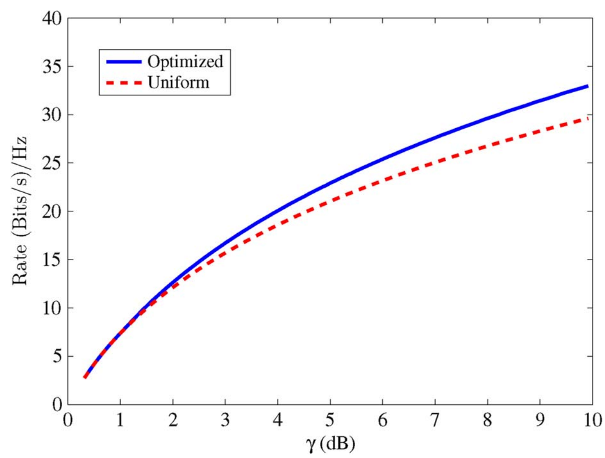

Fig. 6. Transmission rate vs. γ, $c = 0.0001$.

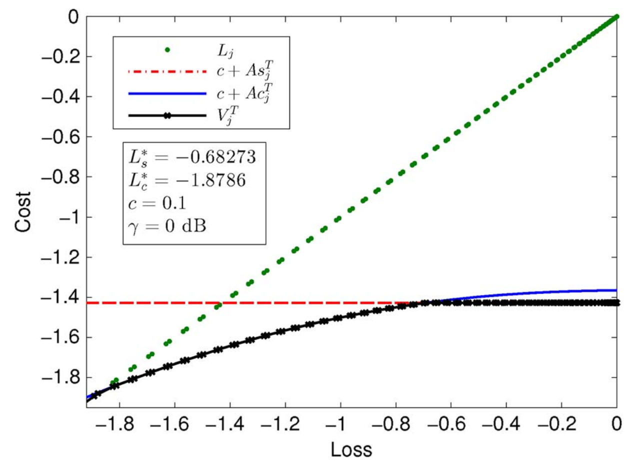

Fig. 7. Cost-to-go function versus loss function.

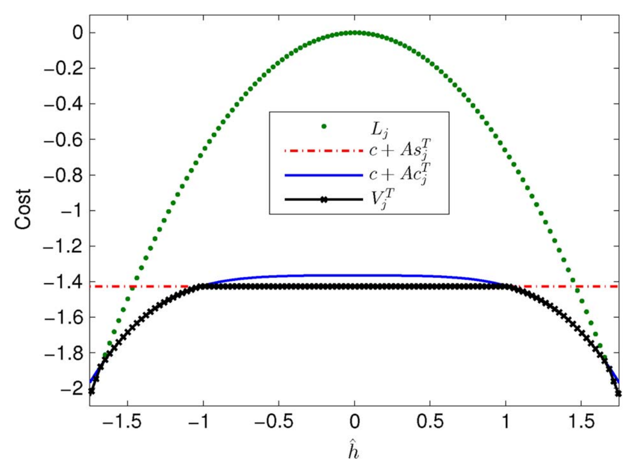

Fig. 8. Cost-to-go function versus channel gain estimate.

different values of γ. Fig. 6 shows the rate versus γ for the same case. The total ASN is significantly reduced when the value of γ increases, while it stays at the same level of αT_t similar to the uniform case before reaching the critical point of $c = \gamma/\rho^2$. However, unlike the uniform case, different channels are optimized to use different values of number of symbols in this region. In *region one*, before the critical point, the rate performances of the uniform and optimized cases are very close, since the optimized scheme requires more time to estimate the channels due to noisy symbols. However, increasing the time, i.e., the value of α, is recommended in the optimized case, but not for the uniform case, since the model optimizes the time for different values of γ. To evaluate the effect of the cost reduction, we now compare Figs. 4 and 6. As the cost increases, the optimized model outperforms the uniform model, while the performance of the optimized model converges to that of the uniform model as the cost decreases. If the cost is zero, the optimized model outperforms the uniform model only when the channels have a broad ranging characteristics. That is, different channels have varying degrees of loss. However, from a practical point of view, the cost should not be zero because as the time spent on the channel training process goes up, the total cost or the transmission loss increases.

B. Numerical Results of the Sequential Good Channel Search Case

To demonstrate the solution, first we will plot the cost-to-go function versus loss. Fig. 7 shows this function along with cost-to-switch and cost-to-continue for $\sigma_w^2 = 1$, $\sigma^2 = 1$, $\gamma = 0$ dB and $c = 0.1$. Notice that when the loss is high, the cost-to-continue is higher than the cost-to-switch and so we have to switch. On the other hand, when the loss becomes smaller than the threshold value L_s^*, the cost-to-continue becomes smaller than the cost-to-switch and so we have to continue until the loss equals the threshold L_c^*. Fig. 8 shows the same cost function versus the estimate $\hat{h}$. This figure demonstrates the concavity of the cost function with respect to $\hat{h}_j$. The threshold values are functions of (c) and γ and we demonstrate this relationship in the following examples.

In Fig. 9, we study the threshold values versus sampling cost (c). In general, the threshold values increase as the cost increases. This is mainly due to the fact that a higher cost per sample yields a faster decision. This will degrade the requirements for the loss and will lead to higher values of the thresholds. Furthermore, the values of L_c^* increase more rapidly, while the increase of L_s^* is very small. The reason for this is that L_s^* depends on the expected value of one sample when we

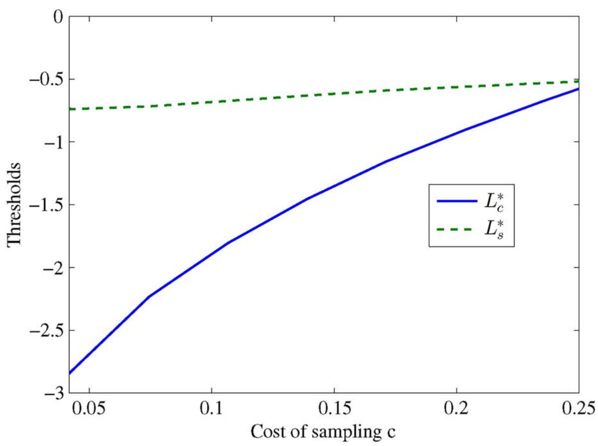

Fig. 9. Threshold vs. cost of sampling c.

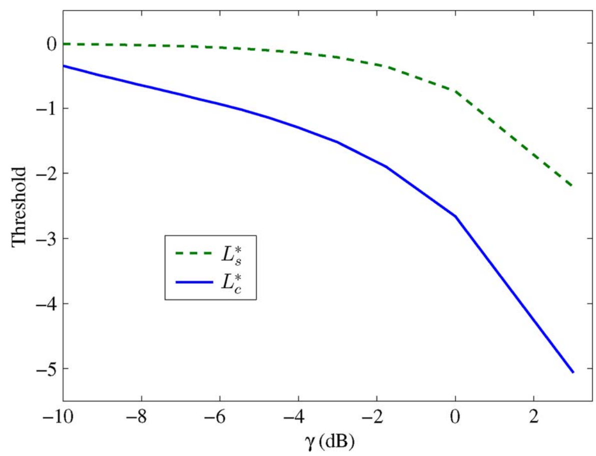

Fig. 10. Threshold vs. γ.

switch. On the other hand, L_c^* depends on a higher number of symbols that makes cost of sampling more effective.

The threshold versus γ relation is shown in Fig. 10 for $c = 0.05$. In general, both values decrease as γ increases, since a higher value of γ increases the requirement to have a smaller loss. The values of L_s^* are close to zero when γ is low and decrease as γ increases since the received symbols become less ambiguous. However, L_c^* decreases more rapidly than L_s^*. This is mainly because for higher values of γ, fewer symbols are needed to reach lower values of the loss.

From the threshold/cost relation we can see that the cost of sampling can relax or tighten the requirements for the channel effective SNR. By choosing an appropriate cost, a better performance can be realized. As for higher sampling cost, the requirements are more relaxed and a good channel could be found faster to balance the higher cost of symbols. However, the more these thresholds are relaxed, the higher the chances are that this channel is not a good one and vice versa. On the other hand, when the cost is low, it may take more time to find a good channel. In this case, due to the low cost, more time is available to seek a good channel. However, the search will require a larger number of symbols. Nevertheless, channel search tests have shown that by choosing suitable values of c, the search almost always yields a good channel.

C. Performance Evaluation

The optimal solution for the multiband sequential estimation problem shows significant improvements over the uniform case. However, to study the performance of the good channel search problem, we will compare it to the optimal solution that explores all the available channels, and then chooses the best one. This case is similar to the optimal solution we found in the multiband sequential estimation case. For the sake of comparison, we will assume the channels are identical and the CR pair estimates all the available channels according to the optimal solution of the multiband estimation case in Section III. The CR pair chooses the best available channel upon completion of the estimation process. The best channel is chosen according to the highest effective SNR defined in (13).

For the good channel case, we will use our optimal model to find the values of the thresholds for given problem constraints. The next step is to benchmark the two solutions in a noisy multiband environment. We will run a Monte-Carlo simulation and find the averages for all the cases. First, define the following variables:

Average Sample Number, Good:

$$ASNG = \overline{\inf\{j : L_j < L_c^*\}},$$

Average Sample Number, Best:

$$ASNB = \alpha_{eff} T_t,$$

Average Good effective SNR:

$$AGSNR_e = \overline{SNR_e^{(k)} | \min(L_j : L_j > L_c^*)},$$

Average Best effective SNR:

$$ABSNR_e = \overline{\max\left(SNR_e^{(1)}, SNR_e^{(2)}, \ldots, SNR_e^{(M)}\right)},$$

Average effective SNR of all channels:

$$ASNR_e = \overline{\frac{\sum_{k=1}^{M} SNR_e^{(k)}}{M}}.$$

Here, the notation $\overline{[.]}$ refers to the average over the number of simulation runs. Table I shows these values for 50 000 Monte-Carlo simulations and $M = 32$. We choose three different cases and compute the defined values for each case. For cases 1 and 2, the value of γ is high and equal, but the cost is lower for case 2. In both cases, it is clear that $AGSNR_e$ is much higher than $ASNR_e$ and a little lower than $ABSNR_e$. At the same time the $ASNG$ is much lower than $ASNB$. When we reduced the value of γ but fixed the cost for case 2 and 3, the $AGSNR_e$ was again much higher than $ASNR_e$ and a little higher than $ABSNR_e$ for case 3. This is not surprising since for $ABSNR_e$, the limited available time have to be divided between all 32 channels equally and we have to explore all the channels. This will not leave enough time to compute good estimates for the channels in a high noise environment, while for $AGSNR_e$, we do not necessarily explore all the channels.

TABLE I
PERFORMANCE EVALUATION

Case	c	γ	L_s^*	L_c^*	$ASNR_e$	$AGSNR_e$	$ABSNR_e$	SNR_e Std Deviation	$ASNG$	$ASNB$
1	0.1	1	−0.68273	−1.8786	0.84565	2.2156	2.7967	1.1464	37	128
2	0.05	1	−0.7392	−2.661	0.86774	2.9758	3.2917	1.1887	47	160
3	0.05	0.1	−0.0144	−0.3485	0.068891	0.37948	0.37844	0.099723	89	448

In both cases, the number of symbols was highly reduced for the sequential good channel search case.

This simulation clearly demonstrates the effectiveness of the optimal threshold rules for searching a good channel. However, some practical considerations should be taken to improve the overall performance. In addition to what we have shown earlier about the effect of the cost and the value of γ in choosing threshold values, some other recommendations could be considered. First, truncation could be used to force a switch in case of highly ambiguous samples when the available time is limited. Second, in order to increase the probability of success in finding a good channel, reasonable threshold values should be used according to the previous analysis we provided. If excessively ambitious values of thresholds are chosen, the model may not find a good channel after exhausting all the available time. To mitigate this, the model can simply use the highest effective SNR that is already found after exhausting all the available time.

VI. CONCLUDING REMARKS

In this paper, we have considered the channel estimation problem in multiband cognitive radio systems. Two scenarios were considered in which the CR users either estimate all the available channels or search for one good channel for transmission. For the multiband estimation case, optimal switching and estimator rules have been derived. The rules minimize a cost function that is a linear combination of the estimation error and the time spent on the estimation. Numerical results have shown that our approach achieves a substantial gain over the strategy that uniformly allocates training sequence to each channel. For the good channel search problem, a sequential procedure was developed to quickly and accurately find a channel with a large gain. It was shown that the optimal switching and termination rules were threshold based rules. These rules were functions of the problem constraints and sampling cost. Numerical analysis were provided to demonstrate the effectiveness of the proposed sequential procedure in finding a good channel.

APPENDIX A
PROOF OF LEMMA 2

Proof: To prove the concavity of $V_j^T(L_j, \hat{h}_j)$, we first want to show that

$$\lambda Ac_j^T\left(L_j', \hat{h}_j'\right) + (1-\lambda)Ac_j^T\left(L_j'', \hat{h}_j''\right) \leq Ac_j^T\left(\lambda L_j' + (1-\lambda)L_j'', \lambda\hat{h}_j' + (1-\lambda)\hat{h}_j''\right), \quad (39)$$

where $\lambda \in [0,1]$, $\{L_j', L_j''\} \in [-\infty, 0]$ and $\{\hat{h}_j', \hat{h}_j''\} \in [-\infty, \infty]$ are two arbitrary points satisfying (22). First, we have $V_T^T(L_T) = L_T$ is concave. To prove (39), assume $V_{j+1}^T(L_{j+1})$ is concave in L_{j+1}, which means

$$\lambda V_{j+1}^T\left(L_{j+1}'\right) + (1-\lambda)V_{j+1}^T\left(L_{j+1}''\right) \leq V_{j+1}^T\left(\lambda L_{j+1}' + (1-\lambda)L_{j+1}''\right). \quad (40)$$

After calculating the LHS of (39) and using the concavity of $V_{j+1}^T(L_{j+1})$ from (40), we have

$$\begin{aligned}
&\lambda \mathbb{E}\left[V_{j+1}^T\left(L_{j+1}'|L_j', \hat{h}_j'\right)\right] + (1-\lambda)\mathbb{E}\left[V_{j+1}^T\left(L_{j+1}''|L_j'', \hat{h}_j''\right)\right] \\
&= \int \lambda V_{j+1}^T\left(L_{j+1}'\right) f_c\left(x_{j+1}|\hat{h}_j'\right) dx_{j+1} \\
&\quad + \int (1-\lambda)V_{j+1}^T\left(L_{j+1}''\right) f_c\left(x_{j+1}|\hat{h}_j''\right) dx_{j+1} \\
&\leq \int V_{j+1}^T\left(L_{j+1}'''\right) f_c\left(x_{j+1}|\lambda\hat{h}_j' + (1-\lambda)\hat{h}_j''\right) dx_{j+1}. \quad (41)
\end{aligned}$$

To compute L_{j+1}''', first define $\hat{h}_j''' = \lambda\hat{h}_j' + (1-\lambda)\hat{h}_j''$ and $L_j''' = \lambda L_j' + (1-\lambda)L_j''$. Then we have,

$$L_{j+1}''' = a_j\left(\lambda L_j' + (1-\lambda)L_j''\right) + b_j\hat{h}_j''' x_{j+1} + d_j x_{j+1}^2,$$

substituting for $\hat{h}_j'''$, L_j', and L_j'' from (29) and simplifying we have

$$L_{j+1}''' = \lambda L_{j+1}' + (1-\lambda)L_{j+1}''. \quad (42)$$

Clearly, the RHS of (39) is equal to

$$\begin{aligned}
&Ac_j^T\left(\lambda L_j' + (1-\lambda)L_j'', \lambda\hat{h}_j' + (1-\lambda)\hat{h}_j''\right) \\
&= \int V_{j+1}^T\left(L_{j+1}'''\right) f_c\left(x_{j+1}|\lambda\hat{h}_j' + (1-\lambda)\hat{h}_j''\right) dx_{j+1}. \quad (43)
\end{aligned}$$

Therefore, we have proved the concavity of Ac_j^T in $(L_j, \hat{h}_j)$, namely

$$\lambda Ac_j^T\left(L_j', \hat{h}_j'\right) + (1-\lambda)Ac_j^T\left(L_j'', \hat{h}_j''\right) \leq Ac_j^T\left(L_j''', \hat{h}_j'''\right). \quad (44)$$

Since $V_j^T(L_j, \hat{h}_j)$ is the minimum of three concave functions, then it is also concave in $(L_j, \hat{h}_j)$. ■

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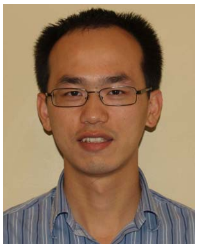

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