Theory and Design of Octave Tunable Filters with Lumped Tuning Elements

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Abstract—This paper presents octave-tunable resonators and filters with surface mounted lumped tuning elements. Detailed theoretical analysis and modeling in terms of tuning range and unloaded quality factor \(Q_u\) are presented in agreement with simulated and measured results. Based on the models, a systematic design method to maximize tuning ratio and optimize \(Q_u\) of the resonator is suggested. A resonator tuning from 0.5 GHz to 1.2 GHz with \(Q_u\) ranging from 84 to 206 is demonstrated using solid state varactors. A two-pole filter with tuning range of 0.5–1.1 GHz with a constant 3-dB fractional bandwidth (FBW) of \(4 \pm 0.1\%\) and insertion loss of 1.67 dB at 1.1 GHz is demonstrated along with a three-pole filter with tuning range of 0.58–1.22 GHz with a constant 3-dB FBW of \(4 \pm 0.2\%\) and insertion loss of 2.05 dB at 1.22 GHz. Measured input third order inter-modulation are better than 17 dBm over the frequency range for the two-pole filter.

Keywords—combline filter, combline resonator, evanescent-mode design, filter design, filters, full-wave simulation, measurement and modeling, modeling, tunable filters, tunable resonators, waveguide filters

I. INTRODUCTION

RECENTLY, there has been a growing interest in tunable RF/microwave filters. The driving parameters for these filters are low loss, wide tuning, low power consumption, small size, fast tuning, high power handling, and ease of fabrication at a low cost. Various demonstrated tunable filters excel in some parameters at the cost of sacrificing other parameters. For example, planer microstrip filters with lumped tuning components are easy to fabricate, but the unloaded quality factor \((Q_u)\) suffers due to the low Q of the planer waveguides [1]–[3]. To achieve higher \(Q_u\) than planer structures, highly loaded 3-D evanescent-mode (EVA) resonators integrated with various types of tuning technologies have shown promising results [4]–[8]. For example, a tunable filter with piezoelectric actuator showed unloaded quality factor \((Q_u)\) of 700–300 at 4.6–2.3 GHz [4]. Two EVA filters with RF-MEMS tuners, one with switched capacitor network and the other with silicon diaphragm, achieved \(Q_u\) of 500–300 at 5.58–4.07 GHz [5] and \(Q_u\) of 1000–300 at 6–24 GHz respectively [8]. While these technologies attain high \(Q_u\), complexity arises in fabrication due to the precise assembly needed to either align the tuners with the EVA cavity’s vertical gap (typically in \(\mu m\)) or insert the RF-MEMS switching network inside the cavity.

To avoid complicated fabrication and yet maintain high \(Q_u\), 3-D cavities are integrated with commercially available surface mount tuning components as an alternative medium between low \(Q_u\) planer structures and high \(Q_u\) 3-D cavities. In [9], packaged RF-MEMS switches mounted on a substrate integrated waveguide (SIW) are used to get \(Q_u\) of 132–93 at 1.6–1.2 GHz. However the tuning range is limited to a few states. RF-MEMS capacitor banks mounted on a combline resonator resulted in \(Q_u\) of 1300–374 at 2.50–2.39 GHz with a limited tuning ratio (TR) of 1.05:1 [10], [11]. A surface ring gap combline resonator structure loaded with solid state varactors reports a \(Q_u\) of 160–40 with limited tuning range of 3.1–2.6 GHz [12], [13].

The authors of the current paper demonstrated a continuous octave tuning substrate-integrated combline resonator with \(Q_u\) of 86–206 and tuning range of 0.5–1.2 GHz using solid state varactors and \(Q_u\) of up to 240 at 6.6 GHz using RF-MEMS varactors [14]. It is the intention of this paper to further investigate this surface ring gap combline cavity. Compared to previous works [12]–[14], this paper presents an in-depth theoretical analysis of the resonant frequency, \(TR\), and \(Q_u\). Effects of the parasitic capacitance of the surface ring gap and surface inductance are considered to show the compromise between tuning ratio \((TR)\) and \(Q_u\). A design method is suggested to maximize \(TR\) and optimize \(Q_u\). This method demonstrates tunable resonators and filters with higher \(Q_u\) and tuning range than the state-of-the-art with similar technologies. A two-pole filter with tuning range of 0.5–1.1 GHz and measured insertion loss of 1.67 dB at 1.1 GHz is demonstrated. This two-pole filter maintains a constant 3-dB fractional bandwidth (FBW) of 4%. A three-pole filter with tuning range of 0.58–1.22 GHz with a constant 3-dB FBW of 4 ± 0.2% and measured insertion loss of 2.05 dB at 1.22 GHz is also demonstrated.

II. SURFACE RING GAP COMBLINE RESONATOR

Figs. 1(a) and (b) schematically compares the cross-section of the typical vertical gap resonator with the cross section of

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the proposed surface ring gap resonator. In the surface ring gap design, the center post is extended to short the bottom and top of the cavity and a ring gap is created on the surface to isolate the center post from the rest of the cavity’s ceiling. To tune the resonant frequency, tuning elements are placed across the gap to vary the capacitance instead of having to change the physical gap. The figures show that the surface ring gap resonator still has the same lumped equivalent circuit model as the vertical gap resonator. The surface ring gap resonator in Fig. 1(b) resembles a shorted coaxial transmission line and is commonly referred to as a combine resonator.

This design has several advantages compared to the vertical gap. First, the structure is not limited to a particular tuning technology but allows for various types of tuning components, such as RF-MEMS or solid state varactors. Second, precise assembly is not needed since the tuning components are surface mounted. This makes high-volume-manufacturing possible and the resonant frequency and tuning range independent of fabrication and assembly tolerance. Third, this structure is easily implemented on a low cost PCB, which can be integrated with other RF components. However, as shown later, this type of tuning may not yield $Q_a$ in the order of 1,000. Consequently, it is appropriate for applications that need $Q_a$ in the order of 100–600 without the added complexity of vertically-aligned tuners.

### III. Theoretical Analysis and Design

#### A. Resonant Frequency

Fig. 2(a) shows the dimensions of the resonator where $h$ is the height of the cavity, $a$ is the radius of the inner post, and $b$ is the radius of the outer conductor. In Fig. 2(b), the resonator is approximated with lumped elements where $L_p$ is the effective inductance of the cavity and capacitance $C_{1v}$ in series with resistance $R_{1v}$ models a varactor. The packaging and assembly, depending on the type of varactor (solid state, RF-MEMS, etc.), can introduce other parasitics. To keep the model generic, package parasitics are excluded and the varactor is assumed to operate well below its self resonance so the impedance resembles a series RC circuit. The conductors of the cavity, such as the inner post, outer wall, and the top and bottom walls, have resistances of $R_a$, $R_b$, and $R_w$ respectively. A simplified model for the resonator is shown in Fig. 2(c) where $C_v$ and $R_v$ are the equivalent capacitance and equivalent resistance of all varactors and $R_c$ is the sum of all other resistors.

For an L-C resonator, the angular resonant frequency $\omega$ is given as $1/\sqrt{LC}$. Thus, from Fig. 2(c), $C_v$ and $L_p$ are needed to find the resonant frequency of the resonator. The capacitance $C_v$ is extracted from the equivalent impedance of the varactors $Z_v$, which is approximated as

$$Z_v = \frac{1}{N} \left( R_{1v} + \frac{1}{j\omega C_{1v}} \right) = R_v + \frac{1}{j\omega C_v}, \quad (1)$$

$$C_v = NC_{1v}, \quad (2)$$

$$R_v = R_{1v} \frac{1}{N}, \quad (3)$$

where $N$ is the number of varactors placed in parallel on the ring gap. Since the resonator resembles a shorted coaxial transmission line, $L_p$ is extracted from the input impedance ($Z_{in}$) of a shorted transmission line

$$Z_{in} = jZ_o \tan(\beta h), \quad (4)$$

where

$$Z_o = \frac{377}{2\pi \sqrt{\varepsilon_r}} \ln(b/a), \quad (5)$$

where $Z_o$ is the characteristic impedance of the coaxial transmission line, $\varepsilon_r$ is the dielectric constant of the material inside the cavity, and $\beta$ is the phase constant. The above equations assume very low cavity losses, which is typically the case for the dimensions considered in this work. It has been shown that near resonance, the reactance of $Z_{in}$ is inductive [15] and the effective inductance is approximated as

$$L_p = \frac{Z_o}{\omega} \tan\left( \frac{\omega h\sqrt{\varepsilon_r}}{c} \right), \quad (6)$$

where $c$ is speed of light. Assuming low losses, resonant frequency $\omega$ can be solved numerically in

$$\omega = \frac{1}{\sqrt{C_v L_{1v}}}. \quad (7)$$

Eqn. (6) shows that $L_p$ depends on $\omega$, but the change is $L_p$ is small (up to 3% for the cavity parameters and frequency range considered in this paper). Thus, neglecting the variation in $L_p$, the change in $\omega$ or tuning ratio $TR$, is determined by the tuning factor, $F_t = C_{1v_{fin}}/C_{1v_{ini}}$, of the varactor

$$TR \approx \frac{1}{\sqrt{\frac{C_{1v_{fin}}}{C_{1v_{ini}}}}} = \sqrt{F_t}. \quad (8)$$
Ansys HFSS [16] is used to compare Eqn. (7) with full-wave simulations with the cavity parameters given in Table I. The surface gap radius \( r \) and surface gap width \( w \) is explained in section III-C. In Fig. 3, resonant frequencies from Eqn. (7) and HFSS simulations are plotted versus \( C_{1v} \) (see Fig. 2(c)) gives

\[
Q_v = \frac{\text{imag}(Z_v)}{\text{real}(Z_v)} = \frac{1}{\omega C_v R_v}. \tag{9}
\]

Substituting Eqn. (7) into Eqn. (9) and neglecting the variation in \( L_p \) gives \( Q_v \) ratio \( QR = Q_{v_{\text{final}}}/Q_{v_{\text{ini}}} \) as

\[
QR \approx \frac{1}{\sqrt{\frac{C_{1v_{\text{final}}}}{C_{1v_{\text{ini}}}}}} = \sqrt{F_1}. \tag{10}
\]

In Fig. 4, \( Q_v \) versus frequency for three different capacitances are plotted. For a fixed capacitance, \( Q_v \) decreases as frequency increases. However, as the varactor is tuned to a lower capacitance, larger \( F_1 \), Eqn. (10) and Eqn. (8) show that both \( Q_v \) and frequency will increase. Fig. 4 shows \( Q_v \) versus frequency as \( C_{1v} \) is tuned from 2.67 pF to 0.63 pF: \( Q_v \) increases from 124 at 0.6 GHz to 248 at 1.2 GHz. In Fig. 5, Eqn. (9) is compared to results from HFSS simulations. To extract \( Q_v \) from simulation, the conductors (post and walls of the cavity) are simulated as perfect conductors so that \( Q_v \) is approximately \( Q_u \). The reason for higher \( Q_v \) in simulation is investigated in section III-C.

Since \( R_c \) is the resistance of the cavity conductors, the series combination of \( R_c \) with \( L_p \) (see Fig. 2(c)) gives

\[
Q_c = \frac{\omega L_p}{R_c}, \tag{11}
\]

which is essentially the quality factor of the cavity with lossless varactors. The sum of \( R_a, R_b, \) and \( R_w \) in Fig. 2(b) is \( R_c \). From [17],

\[
R_a + R_b = R_s \left( \frac{h}{a} + \frac{h}{b} \right), \tag{12}
\]

in which \( R_s \) is the surface resistance. Surface current density on the top and bottom of the cavity varies radially from the outer conductor to the inner post. Thus integrating in the radial direction gives

\[
R_w = \frac{R_s}{2\pi} \ln \left( \frac{b}{a} \right). \tag{13}
\]
Summing Eqns. (12) and (13) gives $R_c$,

$$R_c = R_a + R_b + 2R_w = \frac{R_s}{2\pi} \left( \frac{h}{a} + \frac{h}{b} + 2\ln(b/a) \right)$$  \hspace{1cm} (14)

Substitution Eqns. (6) and (14) into Eqn. (11) gives

$$Q_c = \frac{377\tan\left(\frac{\omega h\sqrt{\varepsilon_r}}{c}\right)\ln(b/a)}{R_s\sqrt{\varepsilon_r}\left(\frac{h}{a} + \frac{h}{b} + 2\ln(b/a)\right)}.$$  \hspace{1cm} (15)

HFSS is used to compare Eqn. (15) with simulation with cavity parameters from Table I. In order to get $Q_c$ only, $R_{1v}$ in simulation is set to zero and $C_{1v}$ is varied from 2.67 pF to 0.63 pF. Fig. 6 shows a plot of theoretical $Q_c$ from Eqn. (15) compared to simulated $Q_c$. The simulated $Q_c$ is about 3–4% lower than the theoretical one. Fig. 6 also shows that radiation from the ring gap has negligible effect by comparing simulation results from a resonator with a shielding cap covering the top. Note that the ring gap width $w$ is about 100–200 times smaller than the wavelength range considered in simulation. Part of the 3–4% loss in simulation comes from the resistance associated with the ring gap which is considered in section III-C and Fig. 12.

The $Q_c$ of a coaxial resonator is optimized when the $b/a$ ratio is 3.6 [18]. This optimum ratio is again verified by plotting Eqn. (15) in Fig. 7. The outer radius and height of a copper air cavity are fixed to $b = 12$ mm and $h = 5$ mm while $b/a$ is varied from 1 to 25 at four fixed frequencies: 0.1 GHz, 0.5 GHz, 1 GHz, and 2 GHz. The optimized $b/a$ ratio is evident at all frequencies.

Using Eqns. (9) and (15) and the series L-C resonator model in Fig. 2(c), $Q_u$ of the resonator is approximated as

$$\frac{1}{Q_u} = \frac{1}{Q_{c}} + \frac{1}{Q_{v}}.$$  \hspace{1cm} (16)

The resonator's $Q_{u}$ is simulated in HFSS with cavity parameters from Table I. Fig. 8 shows that simulation results are larger than theory because $Q_{c}$ was larger in simulation previously (Fig. 5). In this case, $Q_u$ is much larger than $Q_c$ due to the relatively large $R_{1v}$ (Fig. 6 and Fig. 5), so $Q_u \approx Q_v$. In Fig. 8, $Q_u$ is initially limited by $Q_v$ at lower frequencies (larger $C_v$) but starts to exceed $Q_v$ at higher frequencies (smaller $C_v$). The limitations of $Q_u \approx Q_v$ is analyzed in more details in the next section.

C. Surface ring gap capacitance

In Fig. 3, the theoretical resonant frequency based on the lumped model in Fig. 2(c) was up to 9% higher than the simulated results. Part of this difference is due to the surface ring gap capacitance $C_v$. As shown in Fig. 9(a), this capacitance is dependent on the ring radius $r$, gap width $w$, depth of the gap $d$ and the dielectric constant $\varepsilon_r$. Since $C_v$ is in parallel with $C_{c}$, $\omega$ from Eqn. (7) is modified as

$$\omega = \frac{1}{\sqrt{(C_v + C_c)L}},$$  \hspace{1cm} (17)

where $L$ is given by Eqn. (23). The value of $C_v$ can be extracted from simulated $\omega$ at two different $C_v$ values ($C_{v1}$ and $C_{v2}$) for a fixed cavity structure from

$$\frac{\omega_1}{\omega_2} = \sqrt{\frac{(C_{v1} + C_c)}{(C_{v1} + C_c)}},$$  \hspace{1cm} (18)

where $\omega_1$ is the resonant frequency for $C_{v1}$ and $\omega_2$ is the resonant frequency for $C_{v2}$. $C_{c}$ is extracted for simulated data.
from Fig. 3 to be $C_o \approx 0.8 \text{ pF}$. Compared to Fig. 3, when $C_o = 0.8 \text{ pF}$ is included in Eqn. (17), Fig. 10 shows the difference in resonant frequency is within 1% of simulation. The extraction of $C_o$ here assumes that $L$ is constant as $C_v$ or $w$ varies. The variation in $L$ (Eqn. 23) and how to minimize the variation is considered in section III-D.

Since $C_v$ is in parallel with $C_o$, the total electromagnetic energy will be distributed between $C_v$ and $C_o$. Previously, if the conductors were lossless then $Q_v \approx Q_o$. However, if $C_o$ is large enough, then a significant portion of the electromagnetic energy will be stored in $C_o$. Since the $Q$ of $C_o$ is expected to be higher than $Q_v$ (resistance associated with $C_o$ is small compared to $R_{1v}$), $Q_u$ should exceed $Q_v$. This is however accomplished at the cost of reduced tuning range. The equivalent impedance, $Z_{eq}$ in parallel with $Z_v$, is given by

$$\frac{1}{Z_{eq}} = \frac{1}{Z_v} + \frac{1}{Z_{co}},$$

where

$$Z_{co} = R_o + \frac{1}{j\omega C_o}$$

and $R_o$ is the resistance associated with $C_o$ shown in Fig 9(b) and is initially neglected ($R_o \approx 0$). Then $Q$ of this impedance becomes

$$Q_{eq} = \frac{|\text{imag}(Z_{eq})|}{\text{real}(Z_{eq})},$$

and $Q_u$ from Eqn. (16) is modified as

$$\frac{1}{Q_u} = \frac{1}{Q_v} + \frac{1}{Q_{eq}}.$$

Note that if $C_o$ is small and negligible, then Eqn. (22) reduces to Eqn. (16). Fig. 11(a) plots Eqn. (22) and HFSS simulation results showing the effect of $C_o$ on $Q_u$ and tuning range. $C_1v$ is tuned from 2.67 pF to 0.63 pF. As $C_o$ becomes larger, tuning range decreases and $Q_u$ increases. Also plotted in the figure is $Q_v$ at $C_0 = 0.63 \text{ pF}$ (maximum $Q_v$). When $C_o$ is larger, maximum $Q_u$ for each curve is not limited by maximum theoretical $Q_v$ (dotted line) and far exceeds $Q_v$. But as $C_o$ decreases, maximum $Q_u$ for each curve approaches maximum $Q_v$.

In the case of ideal varactors ($R_v = 0$) or when $C_o \gg C_v$, such as when no varactors are mounted in a static surface ring gap resonator [19], $Q_u = Q_v$ if $R_o = 0$. Fig. 9(b) shows
that $R_o$ depends on the two thin layers of metal wall (dark regions labeled as ring gap resistance in figure) that border the surface ring gap. These two thin walls are simulated in HFSS as a perfect conductors (PEC) or $R_o = 0$ while the rest of the cavity surface is still copper and $R_{3v} = 0$ with parameters from Table I. Fig. 12 compares $Q_c$ of this PEC walls simulation with the previous case in Fig. 6 when all of the cavity surface was copper (no PEC or $R_o \neq 0$). As expected, Fig. 12 shows that $Q_c$ with $R_o = 0$ is higher than $Q_c$ with $R_o \neq 0$. Before, in Fig. 6, $Q_c$ was 3–4% lower than theoretical $Q_c$ (Eqn. 15), but with $R_o = 0$, simulation $Q_c$ is within 1% of theoretical $Q_c$.

In Fig. 13, resistance of the varactors $R_{1v} = 0.8 \, \Omega$ is included in HFSS simulation to compare $Q_u$ when $R_o = 0$ and $R_o \neq 0$. Since $R_o$ dominates both $R_c$ and $R_{1v}$, Fig. 13 shows that effects of $R_o$ is negligible for the parameters in Table I. Thus the previous assumption of $R_o \approx 0$ made in Fig. 11 is valid. Also, compared previously to Fig. 8, theory and simulation are in better agreement, within 1%, now that the effects of $C_o$ are included in theory. Moreover, this figure shows that $Q_u$ is limited by $Q_{eq}$, which reduces to $Q_v$ only if $C_o$ is negligible.

In Fig. 11(a), the ring radius was changed to vary $C_o$. Fig. 11(b) shows that $C_o$ is strongly dependent on $r$. $C_o$ increases with increasing $r$ (increasing circumference) since the surface area of the capacitance increases. Even though $C_o$ has an air gap, the dielectric material of the cavity will partly change $C_o$ due to fringing electromagnetic fields. Fig. 14 confirms this by showing the effects of various $\epsilon_r$ on resonant frequency and $TR$. The results in the figure are based on HFSS simulation with cavity parameters from Table I as $C_{1v}$ ranges from $2.67 \, \text{pF}$ to $0.63 \, \text{pF}$. Simulation shows that $TR = 1.93$ when $\epsilon_r = 1$ and $TR = 1.72$ when $\epsilon_r = 10$, a reduction of about 11% in $TR$. Based on Eqn. (18), $C_o \approx 0.8 \, \text{pF}$ for $\epsilon_r = 1$, $C_o \approx 1.75 \, \text{pF}$ for $\epsilon_r = 5$, and $C_o \approx 3 \, \text{pF}$ for $\epsilon_r = 10$.

D. Surface inductance

The current that flows on the top surface from the inner post to the outer cavity wall results in a surface inductance $L_o$. This current flow depends on the ratio of $Z_o$ and $Z_{co}$, and on $N$. If current flow is concentric, $L_v$ will be minimal. The arrangement of the varactors can alter the current path, thus changing $L_v$.

(a) Consider the case of $Z_{co} \gg Z_o$. When $N$ is large and the varactors are spatially distributed, current flows equally in all directions through the varactors and current flow is concentric. Fig. 15(a) illustrates this with $N = 8$. Fig. 15(b) shows current flow when $N$ is small. Current is forced to flow through $N = 1$ varactor resulting in a longer current path. Thus $L_v$ increases as $N$ decreases, which is verified in Table II. In simulation, as $N$ is varied, $C_{1v}$ is also changed to keep $C_v$ constant for different $N$ values.

(b) Consider the case of $Z_{co} \gg Z_v$. Since most of the current is distributed in the surface ring gap, current flow is...
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$Fig. 15. HFSS$ simulation showing surface currents on the varactor loaded surface. (a) Current is equally distributed in 8 varactors and $L_o$ is small. (b) Current flows through just one varactor which increases current path and $L_o$. (c) When $C_o$ is dominant, current flows concentrically through $C_o$ regardless of $N = 8$ or (d) $N = 1$. 

concentric regardless of $N$. Figs. 15(c) and (d) show that current flow is concentric when $N = 8$ or when $N = 1$.

(c) When $Z_o$ and $Z_{cc}$ are comparable, then current is distributed between the varactors and surface ring gap. If $N$ is large and the varactors are equally spaced on the surface ring gap, current flow is concentric. For small $N$, current flow depends on ring radius $r$ and $F_i$. As $r$ gets larger, $C_o$ gets larger and leads to a more concentric current flow, decreasing $L_o$. But as $r$ gets larger, the path of the current flow through the varactor gets longer, increasing $L_o$. Additionally, as $F_i$ is tuned, $C_o$ changes and the impedance ratio of $C_o$ and $C_u$ changes, making $L_o$ vary with $F_i$. Thus, $L_o$ is highly dependent on $r$, $Z_o/Z_{cc}$ ratio, and $F_i$.

The surface inductance $L_o$ is included in Eqn. (17) by replacing $L$ with

$$L = L_p + L_o,$$

resulting in

$$\omega = \frac{1}{\sqrt{(C_u + C_o)(L_p + L_o)}}. \quad (24)$$

Though not discussed earlier, the effects of $L_o$ is included in all the theoretical analysis and plots in section III-C. Even though $L_o$ is dependent on various parameters, based on the above discussion, $L_o$ is always minimal as long as $N$ is sufficiently large. The effects of $L_o$ is further reduced by increasing $L_p$.

As mentioned previously and shown in Eqn. (6), $L_p$ also varies slightly with $\omega$. For $\epsilon_r = 1$, the variation in $L_p$ is less than 0.4% and for $\epsilon_r = 10$, the variation in $L_p$ is less than 3.5% over the frequency range with parameters from Table I.

E. Design Methodology

Based on the theoretical analysis, the compromise in $Q_u$ and tuning range is highly dependent on $C_o$. With the aid of simulation software, appropriate $C_o$ is designed by changing the surface ring gap dimensions or the dielectric constant (Fig. 11(b) and Fig. 14). For example, a resonator is designed for high $Q_u$, 160–40, but with a limited $TR$ of 1.2 in [12]. This is analogous to the curve in Fig. 11 with $C_o = 1.6$ pF which has a $TR$ of 1.22.

Alternately, to design a resonator with maximum tuning range, $C_o$ has to be minimized so that it is small relative to $C_u$. To maximize $Q_u$, $b$ and $h$ can be increased up to the size limitations and $a$ can be set by the optimal $b/a = 3.6$ ratio. Then from (7) and (2), appropriate number of varactors $N$ can be mounted to lower the frequency to the desired range, resulting in an optimized $Q_u$ for a maximum $TR$ design.

This design, however, may result in $N$ that is too large. For example, a cavity with parameters from Table I, except with $a = 3.33$ mm for optimal $b/a$ ratio, requires $N = 20$ varactors on the surface ring gap to tune from 0.6 GHz to 1.2 GHz. If fewer varactors are to be used, then the cavity dimensions may be changed to get the desired frequency range. The height and radius of the cavity are typically limited by the constraints on maximum device size. Varactors with larger capacitances are not recommended since the increase in capacitance comes at the cost of reduced $Q_u$, which directly limits $Q_u$. The reduction in tuning range and cost of high dielectric constant material restricts $\epsilon_r$ as a flexible parameter for design.

The optimum $b/a = 3.6$ ratio may need to be sacrificed to get the desired frequency range. From Eqns. (7) and (6), the resonant frequency depends on the $b/a$ ratio. From Fig. 16, increasing $b/a$ from the optimum ratio of 3.6 to 25 can decrease the frequency from $f_o$ to 0.6$f_o$. The previous example where $N = 20$ varactors were needed to tune from 0.6 GHz to 1.2 GHz can alternatively be designed by increasing the $b/a$ ratio from 3.6 to 24. Fig. 17 shows a reduction of 5% to 7% in $Q_u$ for $N = 8$ compared to $N = 20$.

IV. Experimental Validation

A. Resonator

A substrate-integrated-waveguide (SIW) combline resonator with solid state varactors as tuners is designed and fabricated to validate the theoretical derivations. Fig. 18(a) shows the designed resonator. In this design, metallic vias are inserted in a PCB substrate to create the outer wall of the cavity. The diameter ($\approx 1$ mm) and the spacing of the vias ($< 3$ mm) are designed with the recommendations given in [25] to keep losses minimal. Another center metallic via with radius of $a$ shorts the bottom and top ceiling of the cavity. Fig. 18(b) shows the coplanar-waveguide (cpw) feed lines for this design with length $F_i$. Since solid state varactors are used, a structure
Fig. 16. Plot of $b/a$ ratio versus normalized frequency $f_o$, where $f_o = 1$ is the resonant frequency for the optimum $b/a$ ratio. The resonant frequency decreases by 0.6$f_o$ as $b/a$ is changed to 25.

Fig. 17. A resonator can be designed with $N = 20$ with optimal $b/a$ ratio equal to 3.6. Alternatively, the optimal $b/a$ can be compromised ($b/a = 25$) to design a resonator with the same frequency range where fewer varactors are needed ($N = 8$) with reduced $Q_u$.

with two ring gaps is needed to create a bias point for the varactors. The two rings also allows for back-to-back varactor placement for improved linearity [20]. Since the additional ring is in series with the original ring, Eqs. 2 and Eqn.3 are modified to

$$C_v = \frac{N}{2} C_{1v}, \quad (25)$$

$$R_v = \frac{2}{N} R_{1v}. \quad (26)$$

In order to keep the same $C_v$ and thus the same frequency range, $N$ needs to be doubled for a two ring gap design compared to a one ring gap design.

The first part of the theoretical analysis was based on the lumped model presented in Fig. 2(c). In order for Eqn. (7) and Eqn. (16) to be valid, $C_o$ and $L_o$ had to be negligible. An SIW resonator is fabricated on a Rogers TMM3 substrate with the dimensions given in Table III. By choosing $N = 16$, the effects of $C_o$ and $L_o$ is minimized. Figs. 18(c) and (d) show the fabricated resonator and a close up of the arranged varactors with the isolated bias point for the varactors (not soldered on yet). Fig. 19 compares measured, HFSS, and theory. $Q_u$ is plotted versus frequency as the varactors are biased from 0 V to 30 V. This figure validates that the simplified lumped model from Fig. 2(c) is reasonable in predicting $\omega$ and $Q_u$. As the fabricated $C_o$ and $L_o$ become even smaller, the lumped model should become closer to the measured results. Measured $Q_u$ is lower due to losses in dielectric material (loss tangent is 0.002), fabrication, and assembly.

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<tr>
<th>Cavity parameter</th>
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<tbody>
<tr>
<td>$b$</td>
<td>5 mm</td>
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<tr>
<td>$b$</td>
<td>12 mm</td>
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<tr>
<td>$a$</td>
<td>0.4 mm</td>
</tr>
<tr>
<td>$w$</td>
<td>0.3 mm</td>
</tr>
<tr>
<td>$r$</td>
<td>3 mm</td>
</tr>
<tr>
<td>varactor</td>
<td>Skyworks SMV1405</td>
</tr>
<tr>
<td>$N$</td>
<td>16</td>
</tr>
<tr>
<td>$C_{1v}$</td>
<td>0.63 - 2.67 pF</td>
</tr>
<tr>
<td>$R_{1v}$</td>
<td>0.8 Ω</td>
</tr>
<tr>
<td>substrate</td>
<td>Rogers TMM3</td>
</tr>
<tr>
<td>loss tangent</td>
<td>0.002</td>
</tr>
<tr>
<td>conductor</td>
<td>copper (17.5 µm)</td>
</tr>
</tbody>
</table>
Part of the difference in measured versus theoretical $\omega$ and $Q_u$ is caused by excluding $C_o$ and $L_o$. Fig. 19 plots $Q_u$ versus frequency including the effects of $C_o$ and $L_o$, where $C_o = 0.6$ pF and $L_o = 0.3$ nH were extracted from measured data. The measured $TR$ is about 2.1, simulated $TR$ is about 1.94, and theoretical $TR$ including the effects of $C_v$ is 2.01. The slope of the curve with $C_o$ and $L_o$ is much closer to measured and simulation then the curve without $C_o$ and $L_o$. $Q_u$ increases as more electromagnetic energy gets distributed in $C_o$ as $C_v$ is tuned to a lower capacitance.

A resonator, with similar dimensions as Table III, is fabricated except with $N$ ranging from 1 to 20. To compare the measured $Q_u$ with $Q_v$. Fig. 20 includes a plot of theoretical $Q_v$ at $F_l = 4.2$ (maximum $Q_v$). In the figure, $Q_u$ is limited by $Q_o$ when $N$ is large, but exceeds $Q_o$ for $N = 4$ or less–$C_o$ has become significant compared to $C_v$. In fact, in the extreme case of $N = 1$, $C_v$ is 0.315 pF, which is less than $C_o = 0.6$ pF. When the number of varactors $N$ decreases, $C_o/C_v$ ratio increases and $TR$ decreases. Table IV summarizes the results of frequency range and $TR$. Table IV shows that the $TR$ decreases from 2.17 when $N = 20$ to 1.78 when $N = 1$. Table IV also summarizes the results of $Q_u$ and $Q_o$ ratio at various $N$'s. When $N = 20$ and thus $C_v$, dominates, Eqn. (10) states that $\Delta Q_u = \sqrt{4.2} = 2.05$ which is close to the measured $Q_u$ ratio of 2.38.

**Table IV. SUMMARY OF MEASURED PERFORMANCE OF FABRICATED RESONATORS WITH VARYING N**

<table>
<thead>
<tr>
<th>$N$</th>
<th>freq (GHz)</th>
<th>$TR$</th>
<th>$Q_u$</th>
<th>$Q_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.60–2.84</td>
<td>1.78</td>
<td>35–150</td>
<td>4.24</td>
</tr>
<tr>
<td>2</td>
<td>1.37–2.51</td>
<td>1.83</td>
<td>44–153</td>
<td>3.49</td>
</tr>
<tr>
<td>3</td>
<td>1.18–2.26</td>
<td>1.91</td>
<td>48–163</td>
<td>3.40</td>
</tr>
<tr>
<td>4</td>
<td>1.07–2.09</td>
<td>1.95</td>
<td>51–170</td>
<td>3.33</td>
</tr>
<tr>
<td>8</td>
<td>0.78–1.62</td>
<td>2.08</td>
<td>66–183</td>
<td>2.77</td>
</tr>
<tr>
<td>16</td>
<td>0.58–1.25</td>
<td>2.14</td>
<td>78–196</td>
<td>2.51</td>
</tr>
<tr>
<td>20</td>
<td>0.52–1.12</td>
<td>2.17</td>
<td>90–214</td>
<td>2.38</td>
</tr>
</tbody>
</table>

It should be noted that $Q_o$ of the tunable resonator depends heavily on the tuner technology. In this case, solid state varactors have higher $Q_u$ at lower frequencies. RF-MEMS tuners can be used to get high $Q_u$ at higher frequencies. For example, the authors have demonstrated a surface ring gap resonator with $Q_u$ of 240 at 6.6 GHz [14].

**B. Filter**

To demonstrate the application of the resonator, a two-pole and a three-pole tunable filter are designed and fabricated with the resonator parameters from Table III. A similar procedure as outlined in [21] is used to get the external quality factor $Q_c$ and inter-resonator coupling $K$ from HFSS simulation. HFSS simulation is used to simulate various values of $K$ by changing the width of the coupling iris $w_c$ and the post distance $P_d$ while the feed line $F_1$ is kept far from the post (see Fig. 21(a)). Fig. 22(a) shows $K$ versus $P_d$ when $w_c = 1.6$ mm simulated around 1.1 GHz. A resonator with one port shown in Fig. 21(b) is used to simulated $Q_c$. As seen in the figure, another opening is created at the end of cpw feed lines with a coupling gap size $G_c$ and coupling angle of $Q_{ang}$. Fig. 22(b) shows $Q_c$ versus $Q_{ang}$ when $G_c = 0.7$ mm and $F_1 = 1.35$ cm is simulated around 1.1 GHz.

The required values of $K_c$ and $Q_c$ for a two-pole Butterworth filter response are

$$K_{1.2} = \frac{FBW}{\sqrt{g_{0}g_{1}}} = 0.028,$$

$$Q_c = \frac{g_{0}g_{1}}{FBW} = 35,$$

where $FBW = 4\%$, $g_0 = 1$, $g_1 = 1.4142$, and $g_2 = 1.4142$. Fig. 23(a) and (b) show the designed and fabricated two-pole filter with $w_c = 1.6$ cm, $Q_{ang} = 90$ degrees, and $F_1 = 1.35$ mm in a $5 \times 3 \times 0.5$ cm$^3$ volume. The filter tunes from 0.5 GHz to 1.1 GHz with measured insertion loss from 4.46 to 1.67 dB and measured return loss from 8.3 to 27.8 dB (Fig. 23(c)). A 4 ± 0.1% FBW is maintained throughout the tuning range. An Agilent PNA-X is used to measure input third-order inter-modulation distortion point (IIP3) with the two
Fig. 22. Simulated design curves for (a) $K$ versus post distance $P_d$ and (b) $Q_e$ versus coupling angle $Q_{ang}$.

Fig. 23. (a) Designed and (b) fabricated octave tunable two-pole filter with measured (c) $S_{21}$ and $S_{11}$.

tones separated by 100 kHz. The IIP3 ranged from 17 dBm to 30 dBm when the varactors were biased at 0–30 V. The required values of $K_c$ and $Q_e$ for a three-pole Butterworth filter response are

$$K_{1,2} = K_{2,3} = \frac{FBW}{\sqrt{g_1 g_2}} = 0.0283, \quad (29)$$

$$Q_e = \frac{g_0 g_1}{FBW} = 25, \quad (30)$$

where $FBW = 4\%$, $g_0 = 1$, $g_1 = 1$, and $g_2 = 2$. Fig. 24(a) and (b) show the designed and fabricated three-pole filter with $w_c = 1.6$ cm, $Q_{ang} = 100$ degrees, and $F_l = 1.35$ mm in a $5 \times 3 \times 0.5$ cm$^3$ volume. For this filter, $N = 14$ is used for the resonators to get slightly higher frequency range of 0.58–1.22 GHz with a constant 3-dB FBW of 4 ± 0.2%. Fig. 24(c) shows that the insertion loss varies from 6.2 dB at 0.58 GHz to 2.05 dB at 1.22 GHz.

Since both the two-pole filter and three-pole filter were designed at 1.1 GHz, the simulated $K$ and $Q_e$ values are best matched to the desired values at 1.1 GHz. As seen in Fig. 23(c) and Fig. 24(c), the return loss decreases as the filter is tuned to lower frequencies. In [26], a varactor is mounted on the cpw feed line to change $Q_e$ and another varactor mounted between the two resonators to change $K$. Though [26] had a different resonator technology, the same concept of tunable...
**TABLE V. COMPARISON OF TUNABLE FILTERS**

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Technology</th>
<th>( F_1 )</th>
<th>( TR )</th>
<th>Freq (GHz)</th>
<th>( Q_u )</th>
<th>Fabrication and Integration</th>
</tr>
</thead>
<tbody>
<tr>
<td>[13]</td>
<td>varactors</td>
<td>4</td>
<td>1.09</td>
<td>2.88–2.94</td>
<td>40–160</td>
<td>Simple</td>
</tr>
<tr>
<td>[9]</td>
<td>RF-MEMS</td>
<td>–</td>
<td>1.31</td>
<td>1.2–1.6</td>
<td>93–112</td>
<td>Simple</td>
</tr>
<tr>
<td>[22]</td>
<td>varactors</td>
<td>2.6</td>
<td>1.3</td>
<td>1.7–2.2</td>
<td>( \approx 110^* )</td>
<td>Simple</td>
</tr>
<tr>
<td>[23]</td>
<td>varactors</td>
<td>4.2</td>
<td>1.4</td>
<td>1.4–2.0</td>
<td>( \approx 50^* )</td>
<td>Simple</td>
</tr>
<tr>
<td>[24]</td>
<td>varactors</td>
<td>8.8</td>
<td>1.86</td>
<td>0.7–1.3</td>
<td>( \approx 150^* )</td>
<td>Simple</td>
</tr>
<tr>
<td>This</td>
<td>varactors</td>
<td>4.2</td>
<td>2.2</td>
<td>0.5–1.1</td>
<td>84–206</td>
<td>Simple</td>
</tr>
</tbody>
</table>

*Extracted at highest frequency

\( Q_e \) and \( K \) can be implemented in the presented filter design without added complexity. This allows \( Q_e \) and \( K \) to be tuned as the frequency response of the filter is tuned to improve return loss away from 1.1 GHz. However, adding more varactor will introduce more insertion loss.

Table V compares some of the recent tunable filters. As mentioned earlier and summarized in the table, the complexity of the presented surface ring gap resonator (and other planar varactor tuned filters) is reduced compared to some of the vertically aligned piezoelectric or RF-MEMS based filters. Moreover, to the best of authors’ knowledge, the presented surface ring gap resonator exceeds the \( TR \) of all other solid state varactor based filters for a given varactor capacitance ratio (\( F_1 \)): measured \( TR \) exceeds \( \sqrt{F_1} \) (Eqn. (8)) for this work.

V. CONCLUSION

This paper presents the modeling and design of an octave tunable combline cavity filter using surface mount lumped tuning elements. Detailed theoretical analysis on the tuning range and \( Q_u \) of these resonators/filters are presented. A systematic design methodology is also proposed. To validate the theory and the design procedure, a tunable resonator with \( Q_u \) of 84–206 at 0.5–1.2 GHz, a two-pole tunable filter with tuning range of 0.5–1.1 GHz at a constant FBW of 4 ± 0.1% and measured insertion loss of 1.67 dB at 1.1 GHz, and a three-pole tunable filter with tuning range of 0.58–1.22 GHz at a constant FBW of 4 ± 0.2% and measured insertion loss of 2.05 dB at 1.22 GHz are demonstrated.

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REFERENCES


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